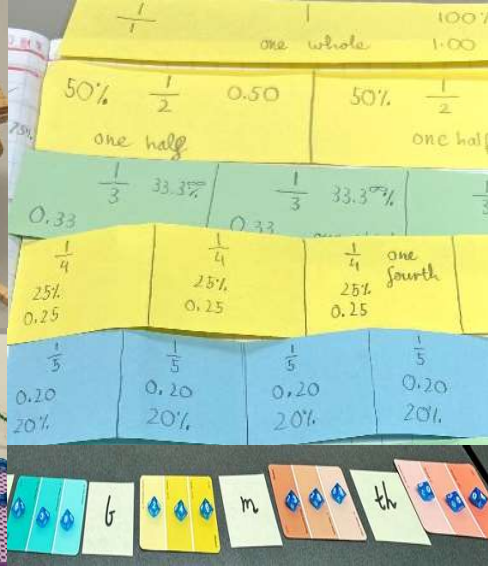


Real-Life Numeracy Years 3-6

Planning Package

Sequential units with hands-on, real-life numeracy for Year 3, Year 4, Year 5 and Year 6 students

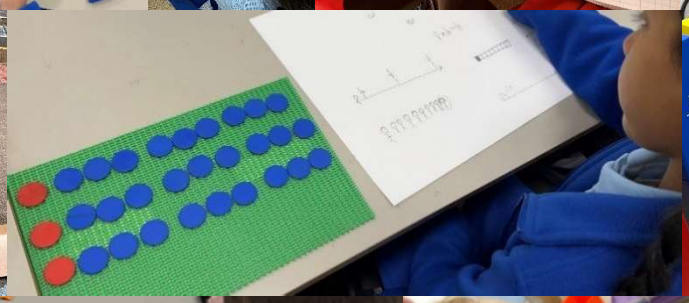
Ten years of development in Australian classrooms.

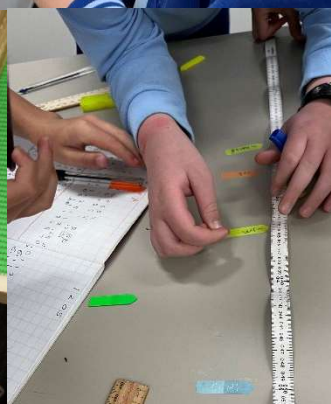
Genuinely high engagement and conceptual understanding in middle to upper primary numeracy.

Comprehensive differentiation for wide ranges: Pre-planned and workable enabling and extending prompts for every lesson.

High-impact, high-relevance professional learning on a daily basis to support planning.

Comprehensive diagnostic and formative assessments to target each sequential point-of-need.





Please note: It is not intended for teachers to attempt to deliver every lesson in this sequence, nor read the unit in full.

Units are designed as **a menu of options**, depending on the points-of-need for each class, with enabling and extending prompts included for every lesson.

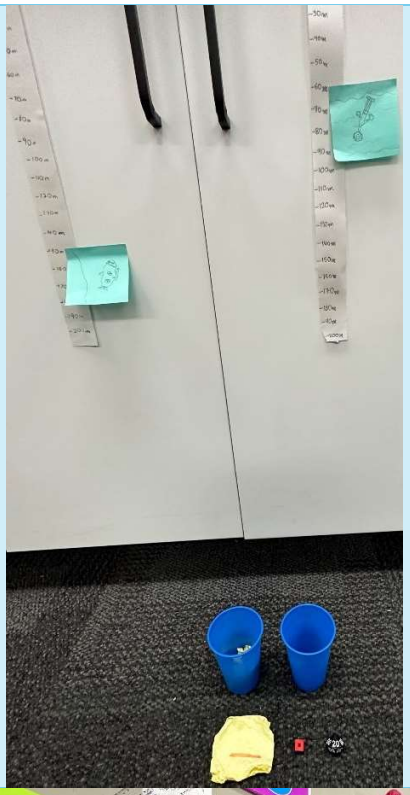
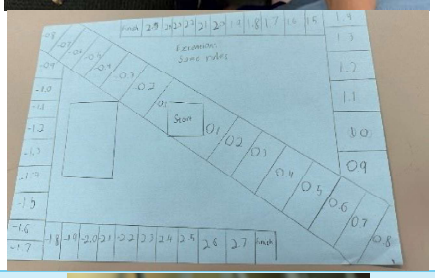
Please choose lesson options based on assessed points-of-need (units are directly linked to the assessments), using either Top Ten's or other **strategy-focused diagnostic pre-assessments.** We recommend avoiding multiple-choice/click-the-answer tests, as numeracy as a discipline grows students' reasoning and thinking skills, ability to explain and show strategies, as well as deep conceptual understanding. Answers alone are not the ultimate goal, or a worthy aspiration in the absence of student reasoning.

Please also select lessons that best suit students' interests and your own creativity and passion. Units are designed to share the wisdom of practice, while respecting and safeguarding the professional role of the teacher as the ultimate best judge of students' needs.

Adjust how many lessons you deliver based on student progress throughout the unit, which can be tracked using the formative assessment folder.

Place Value Unit for Year 6 – 6B Integers

Hyperlinked Table of Contents

Lesson 5 Hot Air Balloon Ride Pages 54-69	 	**Lesson 6** Sea Monster! Deep Sea Divers or Sinking Subs Pages 70-78	
Lesson 7 Mountain Biking Championship Pages 79-86		**Lesson 8** Design Your Own Integers Game Pages 87-93	 
Lesson 9 Saver v. Banker Pages 94-100		**Lesson 10** Integers War! 101-107	
Lesson 11	Mountains and Trenches STEM Task Pages 108-111		

Place Value Unit for Year 6

Curriculum Links for the following lessons

This unit is recommended for Year 6 students.

Australian Curriculum V9 AC9M6N01 and Victorian Curriculum Version 2.0 (VC2M6N01)

Number – Level 6: Recognise situations, including financial contexts, that use integers; locate and represent integers on a number line and as coordinates on the Cartesian plane

- extending the number line in the negative direction to locate and represent integers, recognising the difference in location between (−2) and (+2) and their relationship to zero as $-2 < 0 < 2$
- using integers to represent quantities in financial contexts, including the concept of profit and loss for a planned event
- using horizontal and vertical number lines to represent and find solutions to everyday problems involving locating and ordering integers around zero (for example, elevators, above and below sea level) and distinguishing a location by referencing the 4 quadrants of the Cartesian plane
- recognising that the sign (positive or negative) indicates a direction in relation to zero – for example, 30 metres left of the admin block is (−30) and 20 metres right of the admin block is (+20) – and programming robots to move along a number line that is either horizontal or vertical but not both at the same time
- representing the temperatures of the different planets in the solar system, using a diagram of a thermometer that models a vertical number line

Australian Curriculum V9 AC9M7N09 and Victorian Curriculum Version 2.0 (VC2M7N10)

Number – Year 6: Use mathematical modelling to solve practical problems involving rational numbers and percentages, including financial contexts such as 'best buys'; formulate problems, choosing representations and efficient calculation strategies, designing algorithms and using digital tools as appropriate; interpret and communicate solutions in terms of the situation, justifying choices made about the representation

- **modelling additive situations involving positive and negative quantities; for example, a lift travelling up and down floors in a high-rise apartment where the ground floor is interpreted as zero, or in geography when determining altitude above and below sea level**

Western Australian Curriculum Number and Place Value – Level 6:

Investigate everyday situations that use integers. Locate and represent these numbers on a number line (ACMNA124)

- understanding that integers are …,-3, -2, -1, 0, 1, 2, 3…
- solving everyday additive problems using a number line
- investigating everyday situations that use integers, such as temperatures
- using number lines to position and order integers around zero.

Western Australian Curriculum Number and Place Value – Level 7: Compare, order, add and subtract integers.

NSW Syllabus – Stage 3 – Represent Numbers B

Whole numbers: Locate and represent integers on a number line

- Recognise the location of negative whole numbers in relation to zero and place them on a number line
- Use the term *integers* to describe positive and negative whole numbers and zero
- Interpret integers in everyday contexts
- Recognise that negative whole numbers can result from subtraction (Reasons about quantity)

NSW Syllabus – Stage 4 – Computation with integers A

Compare and order integers

- Recognise and describe the direction and magnitude of integers
- Identify and represent integers on a number line
- Compare the relative value of integers using the less than (<) and greater than (>) symbols
- Order integers

Add and subtract positive and negative integers

- Add and subtract integers with and without the use of digital tools
- Construct a directed number sentence to model a situation
- Examine different meanings (position or operation) for the and signs, depending on context

Extending prompts only: **NSW Syllabus – Stage 4 – Computation with integers A**
Multiply and divide positive and negative integers

- Represent multiples of negative integers as repeated addition
- Multiply and divide positive and negative integers with and without the use of digital tools

Extending prompts only: **NSW Syllabus – Stage 4 – Computation with integers A** Apply the 4 operations to integers

- Apply the 4 operations to integers
- Solve problems involving grouping symbols with integers
- Apply the order of operations to evaluate expressions involving integers, with and without the use of digital tools

Formative Assessment

A formative assessment cross-check is available in this unit's folder with progressive learning goals and specific success criteria for this unit.

Formative Assessment – Ongoing Cross-Check – Integers for Year 6						
Year 6 Place Value Students	Can name at least three real-life contexts for negative numbers **L6**	Can count backwards by ones (0, -1, -2, -3) up to -20 **L6**	Can accurately position negative numbers on a number line (pin the tail on the donkey game) **L6**	Can accurately solve additions and subtractions involving integers with one operator (-3 + 5, or 4 - 6) **L7**	Can accurately solve additions and subtractions of integers with two operators (-5 - - 2, -3 + - 8) **L7**	x and ÷ integers **L8**

Real-life Connections

Real-life examples: It is critical to connect integers to real-life contexts, rather than only a number line. Powerful examples include:

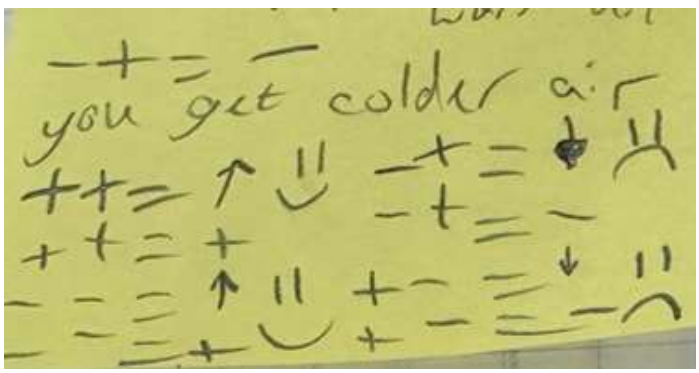
- **Temperatures (positive and negative), particular for schools near the snowfields.**
- Money (saving and owing it to friends/parents, debit and credit).
- Diving (metres below sea level) or elevators (below the ground level).
- Video games and sport, such as golf (score relative to the par) or cricket (particularly indoor cricket, where a loss of wicket results in -5 runs, even if the team has 0).

While it is Year 7 content, many students in Year 6 are ready to start developing the concept of a negative negative, in terms of having two symbols in the one example (such as 5 - - 2).

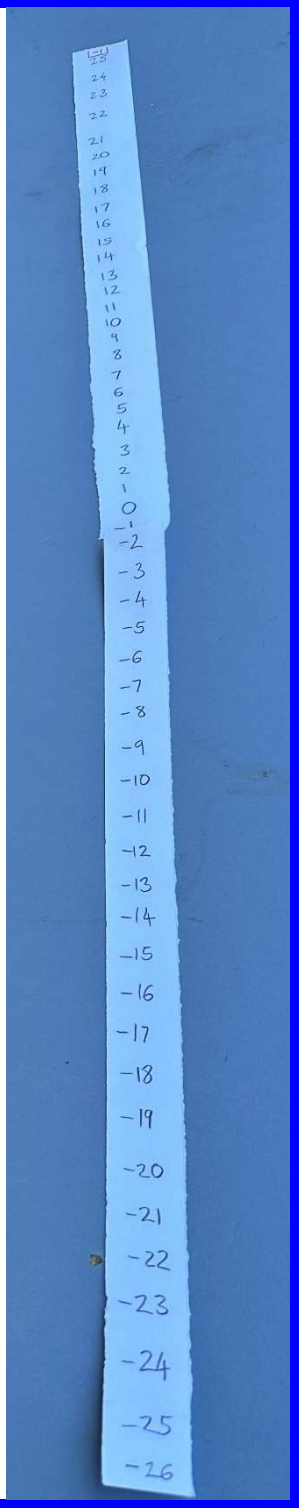
The rules for a negative negative becoming a positive do not always intuitively make sense to students, and so for this reason it is critical to deliver ***The Hot Air Balloon Lesson*** or ***Deep Sea Diver Lesson***, or similar, which contextualise these as adding weight to your scuba pack (adding a negative makes you more negative or heavier) or dropping off sand bags from your balloon (removing a negative makes you lighter or more positive).

Once these concepts are understood with materials following a real-life connection acted out with materials, the following real-life connections can also be established:

- If I take away your puppy (removing a positive from your life, - +), does that make life more positive or more negative? If I give you a puppy (adding a positive to your life, + +), does that make your life more positive or more negative?
- If I take away homework this week and we don't do it at all (removing a negative from life, - -), does that make life more positive or more negative?
- If I take away brussel sprouts from your plate (removing a negative, - -), does that make life more positive or more negative?
- If I give more homework than usual, adding lots to what you have to do (adding a burden or negative element to your week, + -), does that make life more positive or more negative?
- If I add a cookie to your recess snack (adding a positive, + +), does that make life more positive or more negative?



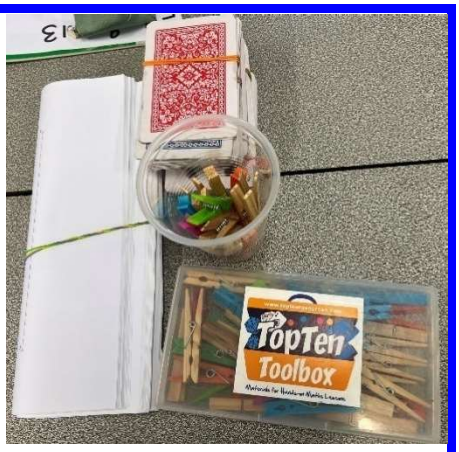
Warm-up Games

Warm-ups	Integers	
Negative Number Scrolls	Students start with a thin strip of A3 paper in front of them. At the top, write 25. Also type 25 into the caculator. Now press -1 = = = using the constant function to continually minus 1. Ask students to work out what comes next by writing it down on their scroll, **before** pressing the equals button for feedback. This ensures the calculator is a feedback mechanism and any mistakes (once students reach zero and beyond), are corrected with immediate feedback – the most powerful learning tool. If it is correct, simply continue. If students get something wrong, they can correct it in red pencil themselves. **When students reach 0, ask, "Can you keep going? Why or why not? Will the calculator allow us to? If so, what will come next?"** After they reach the bottom of their scroll with an interval of -1, make another scroll by -5, -10, -20, -25, -100, -0.1, -0.01 and so on. This ensures students become familiar with the pattern and sequence of negative integers in a very supported way that avoids misconceptions or alerts students to these immediately.	

Pin the Tail on the Donkey (Integers Version)

Focus: Locating integers on a number line

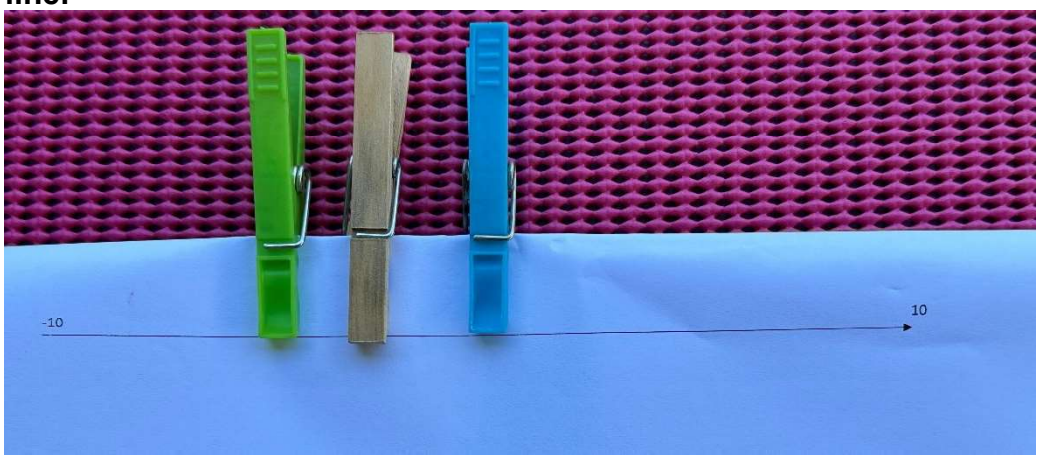
Students use this template with clothes pegs (or paperclips). Fold the template in half, so only one part is showing on each side of the page.



Student A pulls a playing card from a deck. If the card is red, it represents a negative number. For example, a red 3 is -3. Student A then aims to place their peg on -3. Alternatively, both students can place a peg at the same time, then flip it to see who was closer.

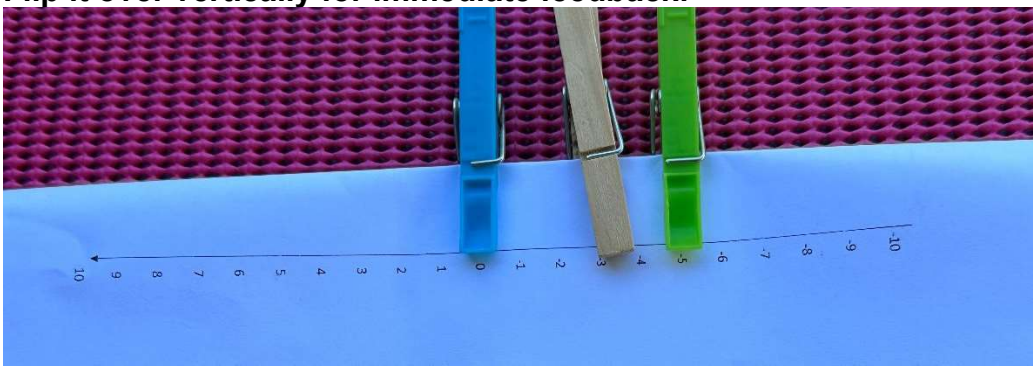
Materials: Template, cards and pegs.

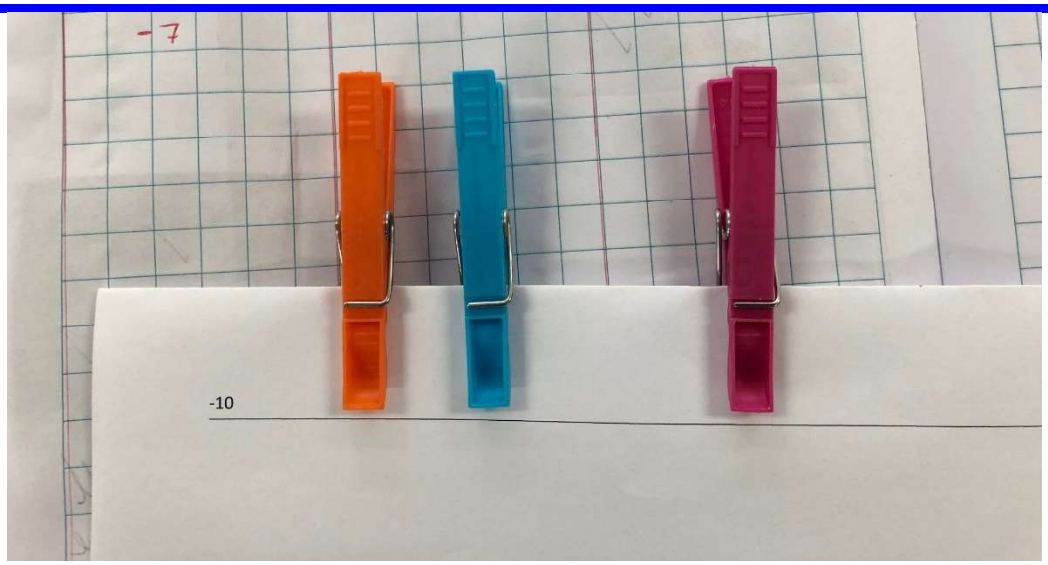
Students flip around the template to receive immediate feedback on the accuracy of their placement of the integer along the number line.



With a target of -3 ('3' playing card in red), here the **coloured pegs are acting as 'benchmark pegs'** for helping numbers such as 0 and -5. Students are also required to verbalise the fractions for these (benchmarking the halfway point, quarter point, three quarter point, and so on).

Flip it over vertically for immediate feedback:

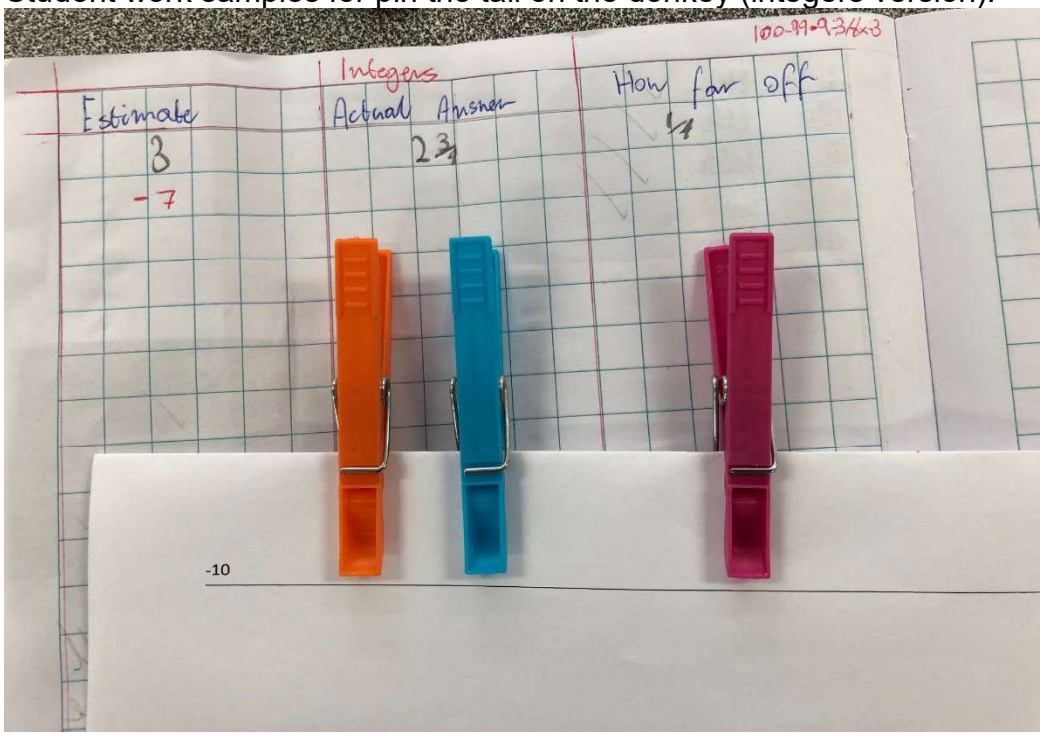




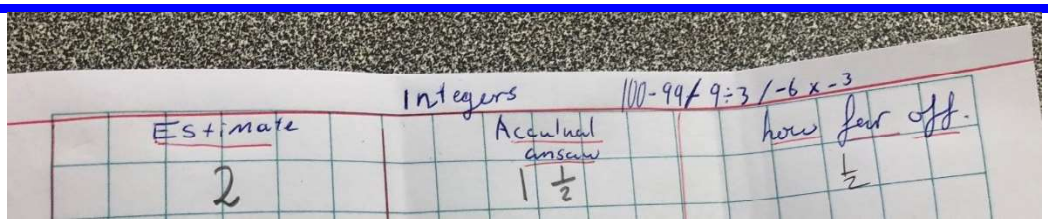
In this photo, the students had pulled a red 7, so were aiming for -7 (orange peg). They were also using a pink and blue peg as their 'guiding pegs' to assist them and think through the location of each integer first. For example, they placed the pink peg at zero, then the blue peg at -5, explaining that they were halving the number line, then halving it again, to narrow down the precise position of -7, which would be just a bit closer to -5 than to -10. **Students record using 3 columns:**

Target	Actual position	How far off?
-7	$-7\frac{1}{4}$	$\frac{1}{4}$

Student work samples for pin the tail on the donkey (integers version):



Students wrote the positive integers in black and negative in red.



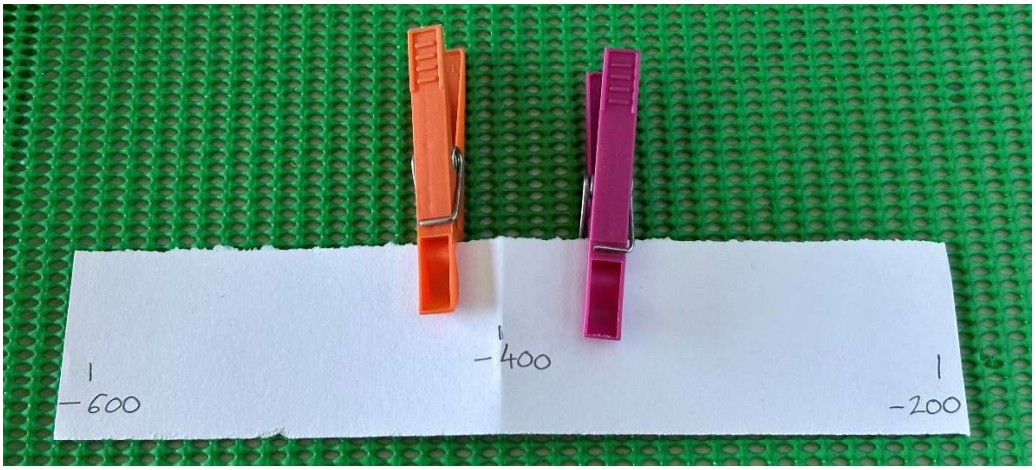
Students also wrote the date using equations (see top right-hand side of the page), rather than just the regular date format of day/month/year.

Extension 1: Use the -100 to +100 pin the tail on the donkey template. There are two ways to create the 'target numbers.' Pull playing cards, but see these as tens. For example, a red 6 would be used to represent -6 tens, or -60 as the target.

Extension 2: Use the -1000 to +1000 pin the tail on the donkey template. Pull playing cards, but see these as hundreds. For example, a red 4 would be used to represent -4 hundreds, or -400 as the target.

Extension 3: Pull 3 playing cards to make a 3-digit number. Student A draws a non-zero start number line where the number could be placed, for example, for -345 (if the cards pulled were '3' '4' '5' and the '3' was 'red' to signify negative) the number line drawn by student A could be -300 to -400, or -200 to -600. Student B and student A then each place a coloured peg where they think -345 would be on this number line.

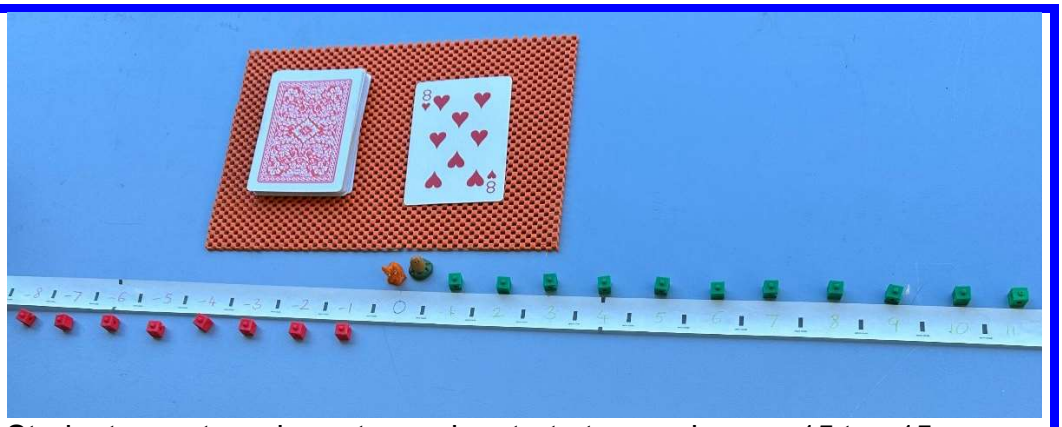
Then students measure their number line in centimetres. Use a calculator to divide the length of the number line by its range. For example, -200 to -600 has a range of 400 (400 is the difference between the maximum and minimum value). If the line measures 15cm, then 15 divided by 400 = 0.0375cm. So, each whole number along the number line is worth 0.0375cm.



Students often have different ideas about which side a number is on, and this is fantastic to start a mathematical debate and uncover misconceptions.

	Multiply this by 145 (345-200, the target number take away the lowest value of the number line), so 0.0375 x 145 = 5.44cm. This is the precise location where -345 should have been placed on their number line. Work out which student got the closest, and repeat. As a simplified example, let's find -154 on a -100 to -200 number line. Let's say the line is 10cm in length. So, 10cm divided by 100 (the difference between or range of -100 to -200) = 0.1cm. Each number is worth 0.1cm or 1mm. So, -154 should be at 0.1x54 (154 – 100 = 54), which means it should be placed at the 5.4cm mark of the line, just beyond the halfway mark and therefore a bit closer to -200 than -100.
Songs and Videos about Integers **Focus: Real-life contexts for integers**	Introduction to integers with teenage actors explaining integers in the context of likes on YouTube and lending your friend money: https://www.youtube.com/watch?v=x0E4vxLydNY&ab_channel=PBSMathClub  Then a quiz by the same teenage actors: https://www.youtube.com/watch?v=aFvZEEfI3hU&ab_channel=PBSMathClub  Integers song: https://www.youtube.com/watch?v=u69pYSdwugo&ab_channel=MathSongsbyNUMBEROCK

Collect the Integer Points



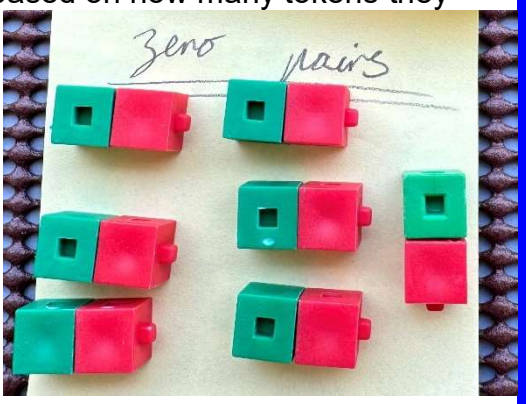
Students use two characters, who start at zero along a -15 to +15 number line. For a link to measurement and to show the concept that negative integers are an equal distance from 0 as are their positive integer counterparts, students could draw the number line themselves, measuring it out on a 30cm piece of paper, starting with 0cm in the centre.

Students place a 'points token' at each integer along the line. Students then take turns to pull playing cards. A red card makes their character move backwards; a black moves their character forwards. If they go off the end of the line (below -15 or above +15, they miss that turn – do not collect any tokens for that turn – then reappear at 0 the following turn).

Wherever they land (their final spot after they move their character – not each box along the way), they collect that token. The first player to collect 16 tokens, wins!

Extra engagement: Students can act like Mario, making a sound effect when they collect each point!

Extension: Negative/red numbers could have red tokens and positive/green numbers could have green, with no token on zero as it is neither a positive nor a negative number. At the end of the game, students total their score, not purely based on how many tokens they successfully collected, but on the mix of red/green in their hand. For example, if they have 4 red/negative and 6 green positive, their score would be +2, as the 4 negative cancel out (zero pairs) 4 of the positive. This can be solved by connecting the reds to greens (if using connectable cubes), as shown.



Integers
Year 6
Lesson 4

Party in Paradise or Walk the Plank

Learning intention: Recognise the position of integers on a number line. Add and subtract integers using a pre-filled number line. Compare integers using their position on the same number line.
Maths vocabulary: integers, the origin (zero), negative, positive, directed number sentence, direction, magnitude, additive inverse

Connect to students' interests: Ask students to brainstorm their absolute favourite places to be and places they dislike with a passion. Students can draw their favourite place on one post-it note and their least favourite on another. If students are struggling, provide inspiration for some amazing places, such as hotels in the Maldives, peaceful spots in botanical gardens, and an awesome water park. Their favourite

Lesson summary: Students start with a -25 to +25 (or similar) number line on their desks, made from strips of paper. Students play against a partner, using a mini figurine as their player, starting at the origin (zero). Students pull playing cards (red is negative – move backwards, black is positive – move forwards), aiming to reach +25 and avoid -25.

Materials:

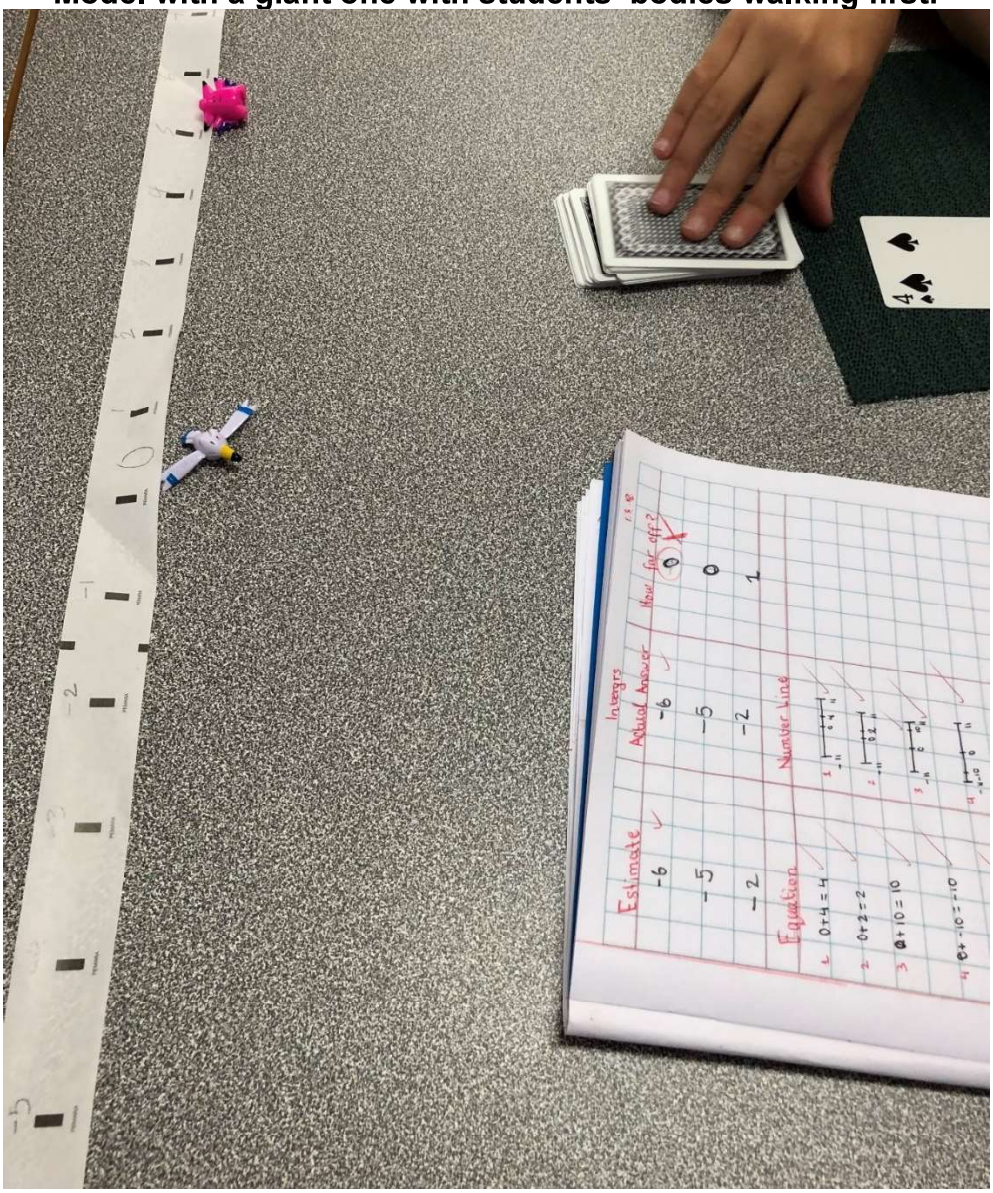
- Thin strips of A3 paper (pre-sliced by the teacher) for students to create a number line that spans the width of a partner desk (about 1m or 1.5m) by sticking these together. Students work out the halfway mark, writing '0' at this point. Students then measure intervals of 3cm, writing the negative numbers to the left in red pencil, and positive to the right of zero using black pencil. **Retain for future lessons in this unit.**
- Playing cards (regular deck with red and black cards, because these colours will be used to represent negative and positive). Remove all picture cards. Red cards = negative. Black cards = positive.
- Recording template available, or students can record in their books using 3 columns (outlined in the next pages).
- Mini figurines – 1 per student, such as Pokémon counters or Lego men.

Best set-up: Fishbowl model playing against a student, then play in like-pairs on mixed-ability tables. Attempt extension 1 for advanced students (and return to this extension option later in the unit for mid-range students).

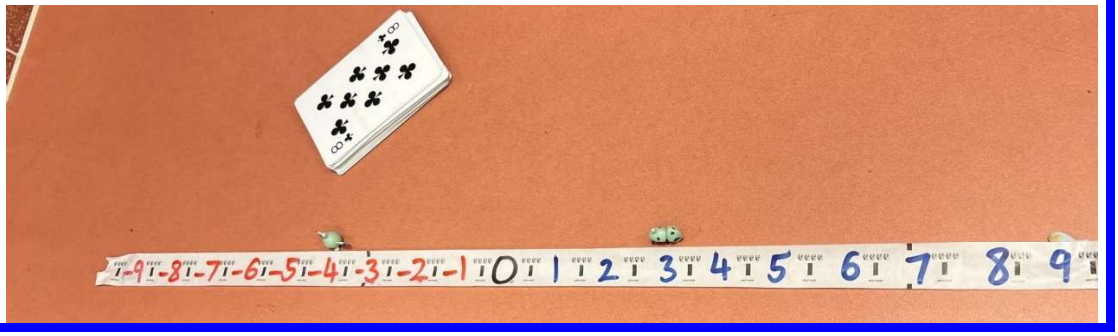
place post-it note then goes at the positive end of their number line for the game, and their least favourite or place they wish to avoid most goes at the negative end.

Whole-class discussion: Reflect on the learning from lesson 1 by revising some real-life examples of negative numbers. Show students any negative number dioramas/ displays from lesson 1 that represented personal best work by the students who created them, or any that were particularly creative and worthy of being highlighted.

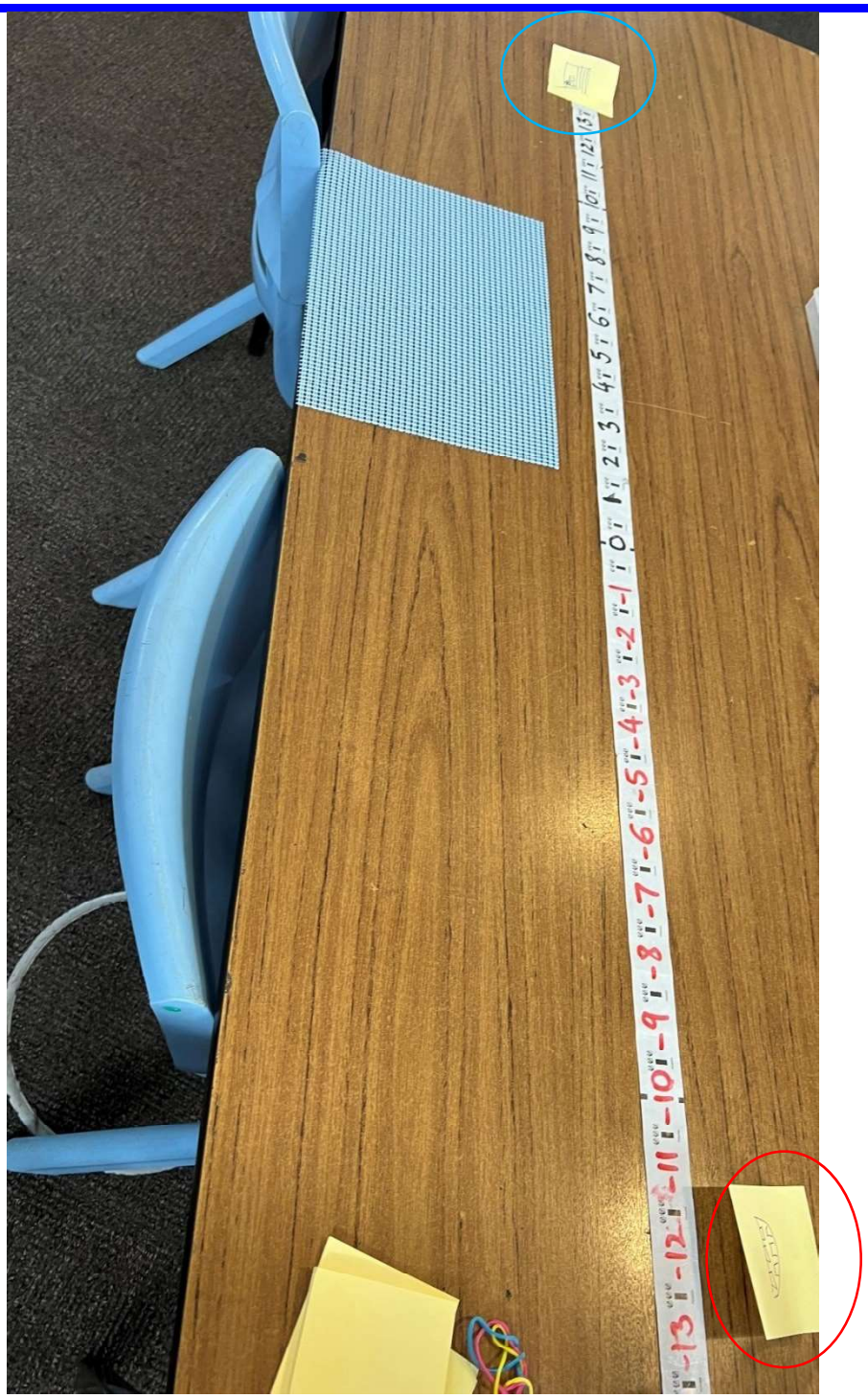
Model with a giant one with students' bodies walking first!



Lesson in action



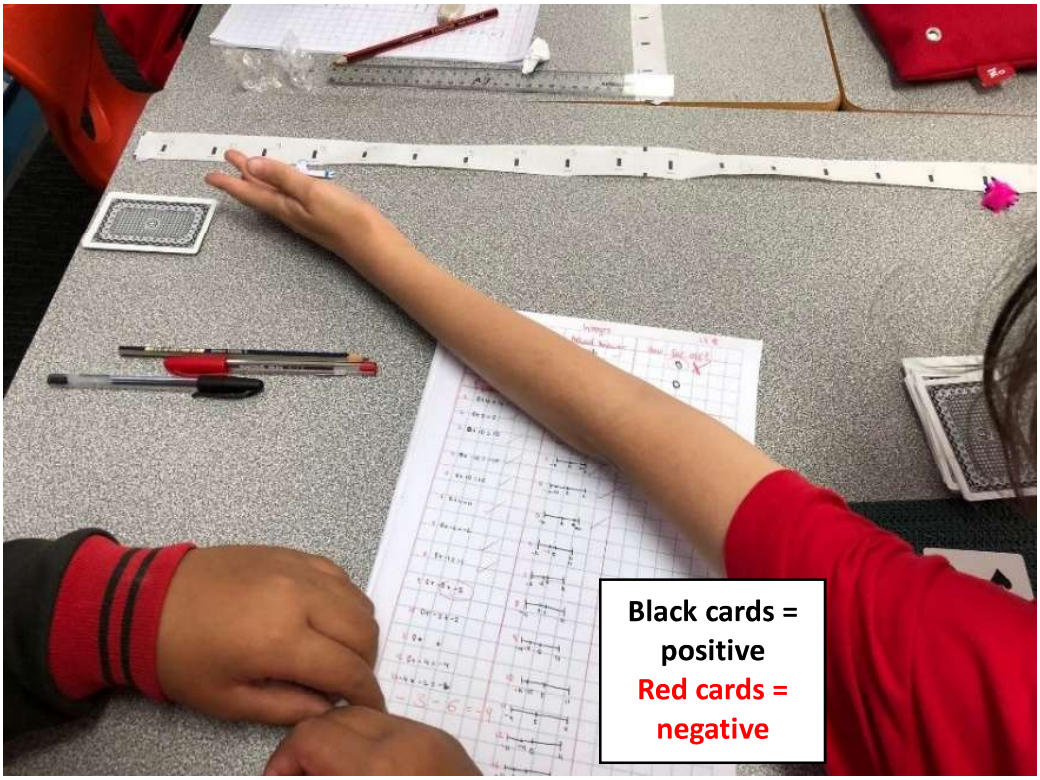
Part of getting to the end of primary school is not just about learning really large numbers like millions, billions and trillions, but also learning about really tiny numbers (like decimals that live between each whole number and fractions that split items or numbers into parts). It is also about learning what numbers come before zero, and how these work.



Fishbowl modelling set-up with post-it notes for a student's example of their favourite place (paradise at the end of positive section – here it was the cinema) and least favourite place (plank at the end of the negative section – here it was the dentist)

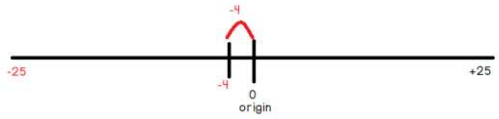
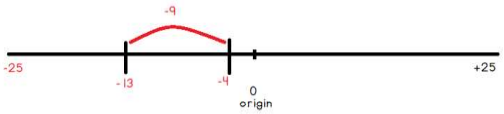
Note: Fishbowl set-up is always inverted to ensure most students receive a good view, so the teacher works upside down with a student partner.

Modelling: Start with students creating number lines to stick along the width of their desk, using long strips of pre-cut slices of A3 paper. Students first create a length of about 150cm, work out the halfway mark, writing '0' at this point. Emphasise that zero is called **the origin** and ask students to mark this with a post-it note to highlight this vocabulary. Students then measure intervals of 2cm or 3cm, writing the negative numbers to the left of zero in red pencil, and positive numbers to the right of zero using black pencil. Students place their 'favourite place post-it note' drawing from the hook (see left column), at the highest positive number on their number line (+25). Students place their drawing of their least favourite place at the lowest negative number (-25). This is where the name of the game comes from – if students reach the positive end of the number line, their character parties in paradise, but if they fall off the negative end, they have walked the plank to doom!



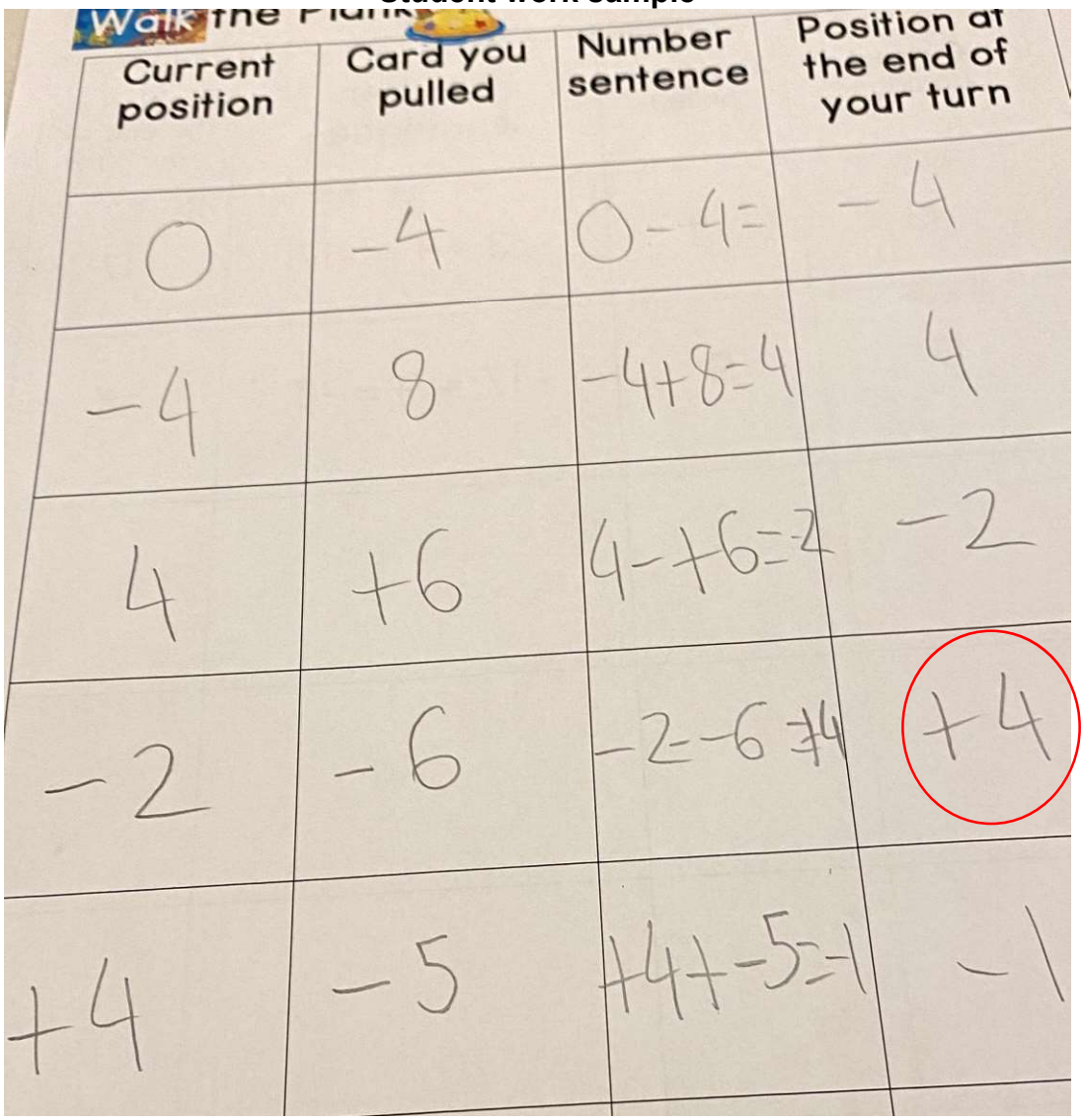
Students start their characters at the origin (0). Students take turns to pull cards. Their character always starts their turn by facing forward – towards the positive numbers. If the card is red, the character moves backwards, further into the negative numbers. If the card is black, the character moves forward, to the positive end. The colour of the card shows the **direction** (positive or negative), and the quantity on the card shows the **magnitude** (size). *NZ Maths:* **Integers** are sometime called directional numbers because they represent a **magnitude** (size) and a **direction**, e.g. $^{-}3$ represents a **magnitude** of 3 units in a negative **direction** from zero.

Students record using two columns:

Directed number sentence (a number sentence that includes integers)	Number line
0 – 4 = -4	
-4 – 9 = - 13	

There is also a recording template available in this unit's folder.

Student work sample

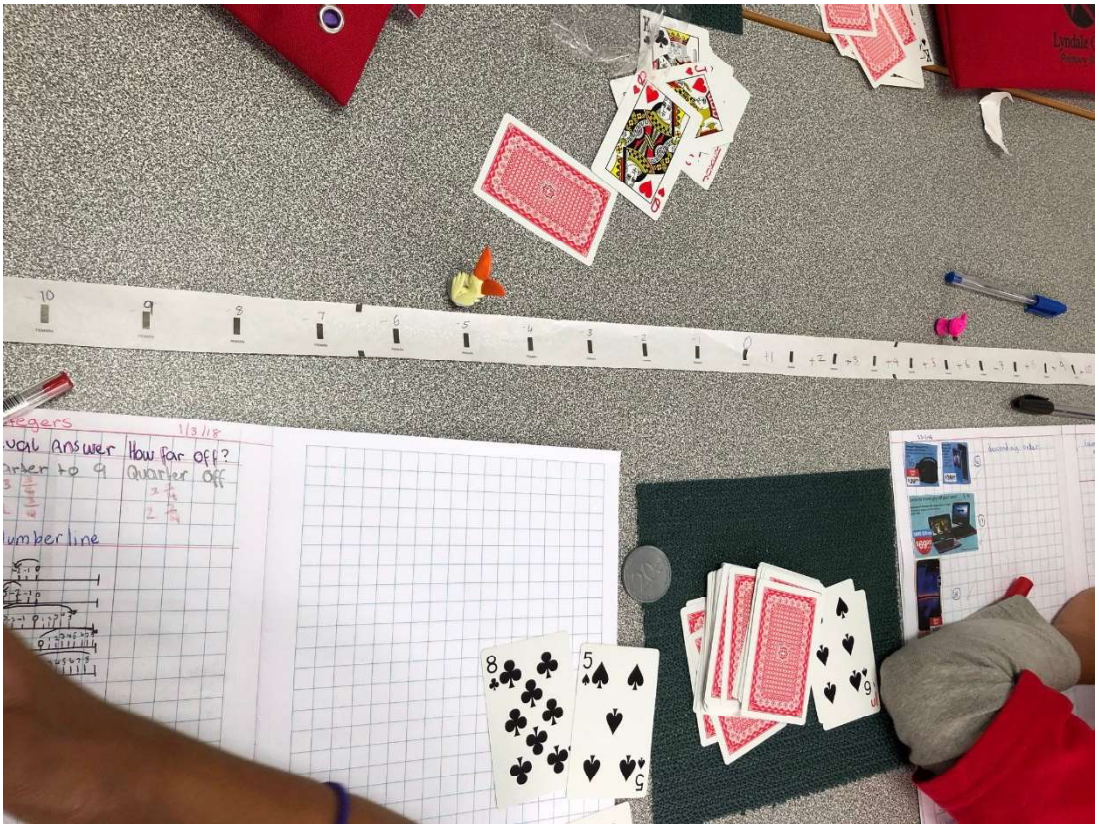


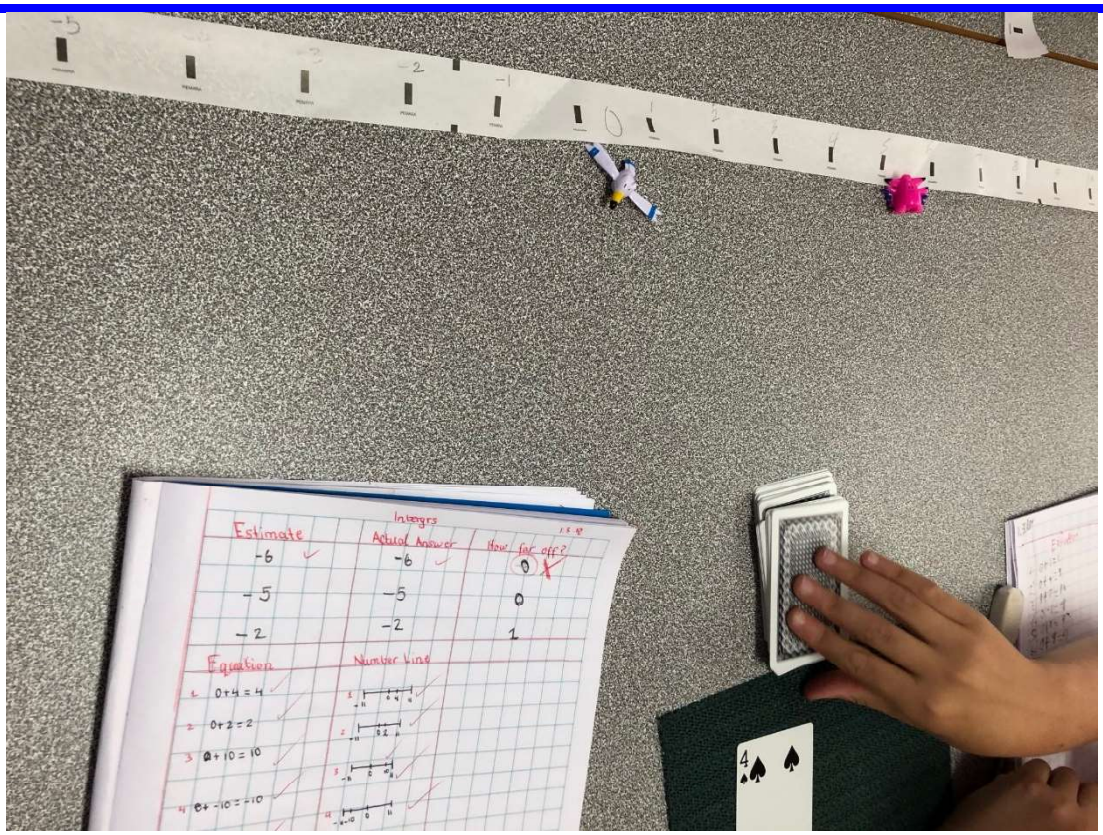
Current position	Card you pulled	Number sentence	Position at the end of your turn
0	-4	0 - 4 =	- 4
-4	8	-4 + 8 = 4	4
4	+6	4 - +6 = -2	-2
-2	- 6	-2 = -6 = +4	+4
+4	- 5	+4 + -5 = -1	- 1

During the COVID remote learning periods, Top Ten created a home learning video of this game with a year 6 student available at:
Integers Number Line – Walk the Plank – YouTube
(https://www.youtube.com/watch?v=pnrF3TQ8jx)

A slideshow explanation of this task is available, which was also created for the 2020 remote learning periods.

Lesson in action in Year 6





Questioning:

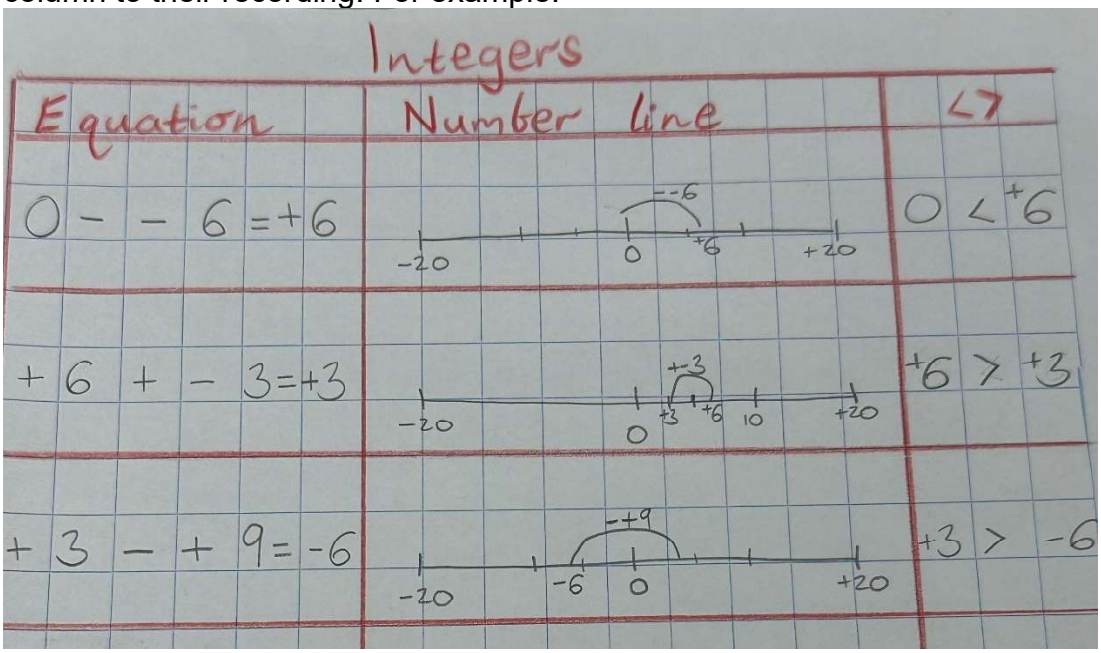
- Do you have to write '+4' for positive numbers, or is just writing '4' okay? (You have been learning about positive numbers for years, so just '4' means the same thing of course, but if you want to make it clear while working with negative numbers at the same time, you can write '+4).
- Can fractions be positive or negative? Where would $-\frac{1}{2}$ be placed on this number line? What about -5¼? Do negative decimals exist?
- If you are at -8, what number is the same distance away from zero? (+8), it is the **additive inverse**, meaning if you add 8 to -8 the answer becomes zero, because the +8 moves the -8 back to the origin. What is the additive inverse of -50? What about -227?
- If you are at -4 and you pull a black 4 (+4) where do you end up? (Zero). *Hint:* Remember that -4 is the **additive inverse** of +4.

End-of-session challenge: Flip over your line number so it is blank again. Can you make a number line that goes from -1000 to +1000? Or -1500 to +1500? Does -1000 and -1500 exist? How can you be sure? What about -10 000, or -100 000?

Research the average mortgage in your state – yes, those negative numbers definitely do exist and are very relevant to many people's lives who own houses and are currently paying them off.

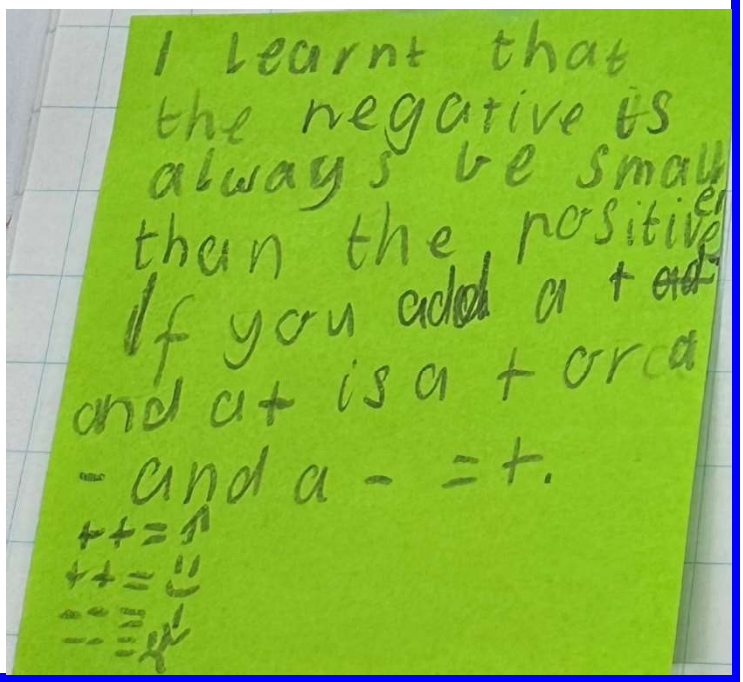
Second repeat session challenge – comparing integers

Repeat session challenge: Run a second/repeat lesson of this, however, require that students compare their integers at each turn, adding a third column to their recording. For example:



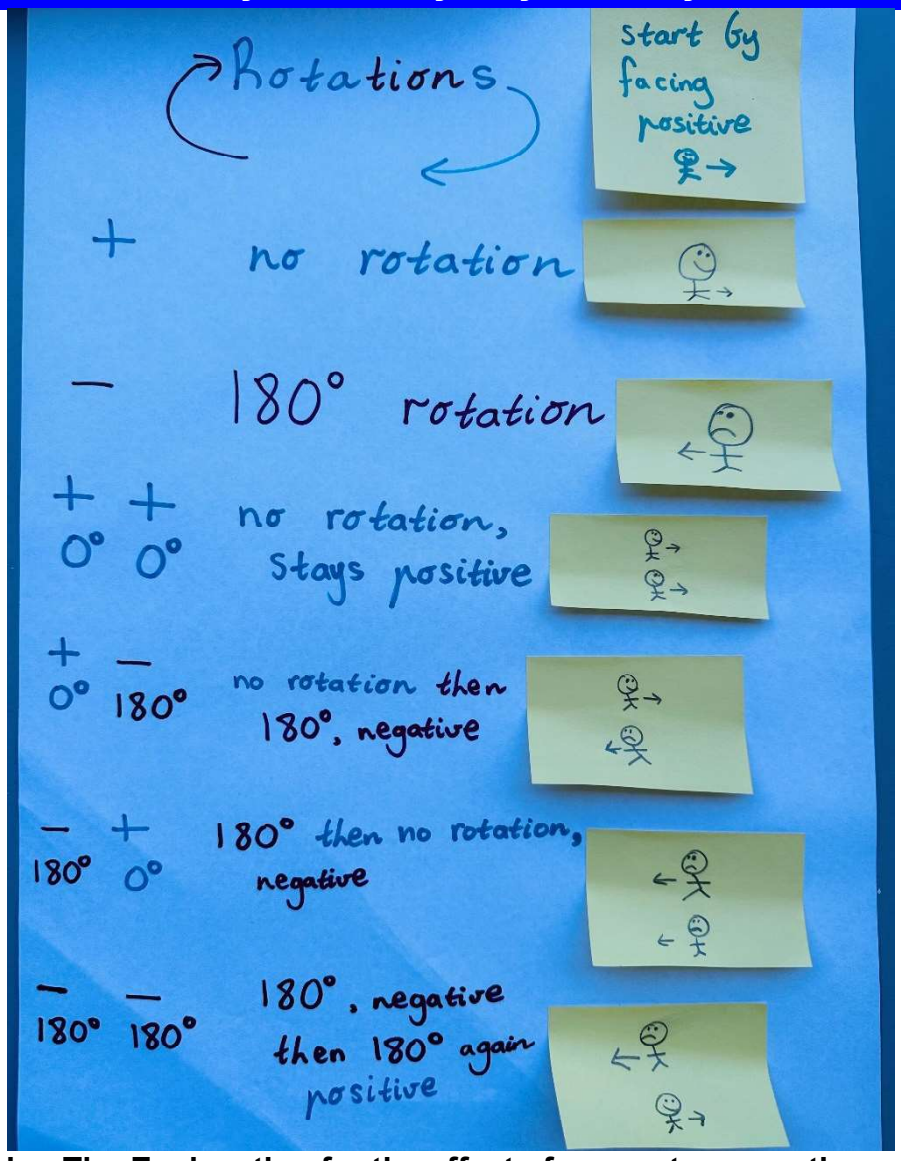
Common misconception alert:
Students may believe that -8 is bigger than -4. -4 is in fact worth more, because it is more positive than -8. -8 is further towards the negative or 'evil/underworld' of numbers.

Reflection post-it note:



Support: Clean their deck of cards so that it only contains 1-4 (not 1-9). This reduces the mental computation difficulties when mixed with the new concept of negative numbers. Also reduce the range of their number lines to -12 and +12, so the games are shorter and move quicker for more engagement. This also makes the student recording (number line recording column) significantly more supported.

Third repeat session: Year 7 content option, which can be attempted by extension students during session 1, and other students when they show they may be ready



Teaching Tip: Explanation for the effect of one or two negatives: Each negative has the effect of rotating your character 180 degrees. This links to later secondary school concepts, where this geometric intuition will benefit students greatly when they study complex numbers.

With this in mind, this would be the effect of two consecutive symbols:
+ + = + because the '+' sign does not cause any rotation, only the '-' sign because it turns you backwards 180°
+ - = - and -+ = - because the '+' does not rotate your character, but then the '-' does, resulting in a 180° rotation and your character moving backwards
- - = + because if you rotate 180° twice you end up facing forward again.

Integers
Year 6
Lesson 5

Hot Air Balloon Ride

Learning intention: Add and subtract positive and negative integers, using directed number sentences that include both direction and operation symbols. *Support option:* Move in both directions on an integers number line.

Maths vocabulary: integers, positive, negative, add, subtract, compare, direction, magnitude, directed number sentence

YouTube hook: Time lapse by National Geographic of hundreds of balloons taking flight in New Mexico https://www.youtube.com/watch?v=QGAMTII6XxY&ab_channel=NationalGeographic

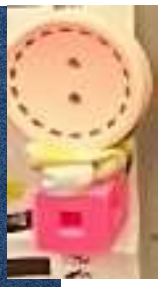
YouTube hook: Virtual Reality hot air balloon ride: https://www.youtube.com/watch?v=2KT0rn03-Tu&ab_channel=Hallmark

STEM: Read about how hot air balloons work: https://napavalleyballoons.com/how-

Lesson summary: Students act out a hot air balloon ride, adding and subtracting integers represented by sandbags (negatives) that sink their balloon, as well as hot air that takes their balloon further up into the sky (positives). VIDEO OF THE LESSON IN ACTION

Easter Egg balloons.

Balloons constructed using buttons and cubes.



'+ add' written on one post-it note
'- take away' written on the other.

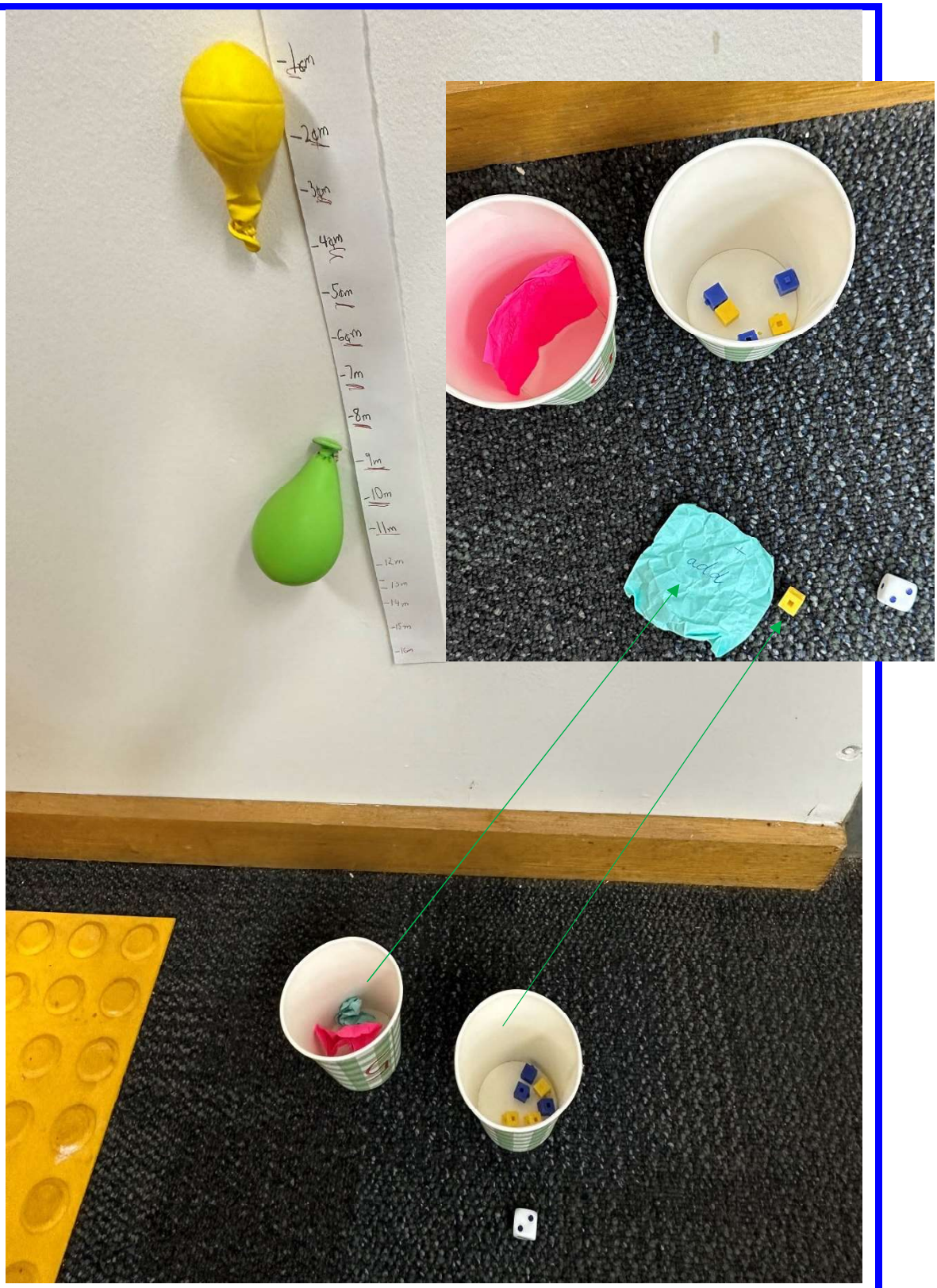
does-a-hot-air-balloon-work.php#:~:text=Science%20tell%20us%20that%20hot,causing%20it%20to%20slowly%20descend.

Materials:

- Craft materials to represent hot air balloons – these could be Easter Eggs, buttons with cubes Blu-tacked beneath, or actual balloons that students only blow up by the tinniest amount with Blu-tack behind them.
- Thin strips of A3 paper for students to create a number line with labels from about -15 to +15, or -20 to +20, or similar. Students work out the halfway mark, writing '0' at this point in black. Students then measure intervals of 2cm, writing the negative numbers below the zero in red pencil, and positive above the zero using blue. This could be extended to -40 to +40 (or similar) to build on the progress from the previous session. The integers number line is set up vertically (not horizontally as in the previous session) and stuck to a vertical surface like a desk leg or wall.
- Cubes – blue as the air (positives) and yellow as the sandbags (negatives).
- Post-it notes – 2 per pair. Write 'add/+' on one and 'take away/-' on another.
- 2 non-transparent cups. Students place the cubes in one cup, and the post-it notes in another, pulling the elements for each round.
- 3-dot or 6-sided dice to determine the quantity of air or sandbags to imagine adding or subtracting from their balloon.
- *For support version:* Playing cards.

Best set-up: Whole-class model at a desk with materials. Students can work individually, or as a team, or compete to reach the positive end before their partner, or another team, (if either falls off the negative end, they lose).

Set the context for the game with a story: "You were gliding along during your first ride in a hot air balloon, but the pilot started to feel sick. He faints on the floor! Now it is up to you to guide the balloon to safety. The balloon is flying over a river. With the pilot down, the balloon has gone so low that it has started skimming the water. It is currently 6cm below the surface of the water. If it reaches -25cm (the end of the negative number side), it loses all flight and sinks! If it reaches +25cm, it has recovered and you can sail safely away – you've saved the day! Start at 0cm, skimming the lake's surface, and try to get to +25cm before your partner does in their balloon – who will save their balloon first?



2 cups at the ready – one with scrunched-up post-it notes reading ‘add’ and ‘take away’; the other with blue cubes (air) and yellow cubes (sandbags) that get replaced each turn, and a 3-dot dice.

Modelling: Students start their balloon at **the origin** (0). Students then pull the game elements out of the cups (a cube and a post-it note).

1. First, pull one post-it note from the post-it notes cup. Pulling a post-it note that says 'add' means the students adds an element to their balloon. Conversely, pulling a 'take away' means the students take away that element from their balloon.

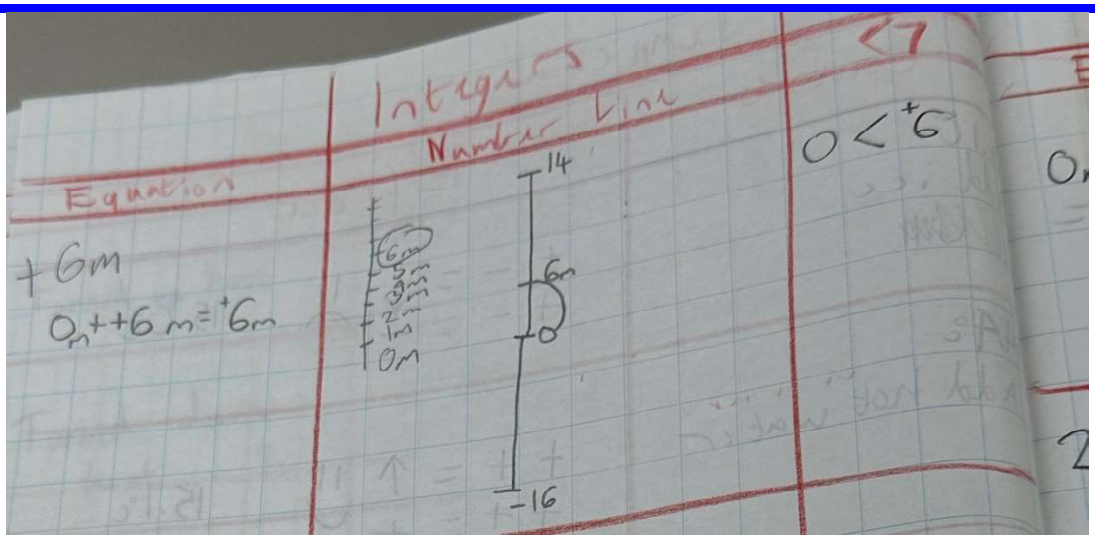
2. Next pull from the cubes cup. Pulling a blue cube means air and is a positive (enabling the balloon to fly higher with more hot air in it, if it is added, but deflating the balloon if it is subtracted). Pulling a yellow cube means sandbags, as a negative, since it can make the balloon heavier if added, or lighter if subtracted.

3. Finally, students roll the dice to see how many places, along the vertical number line, their balloon should move.

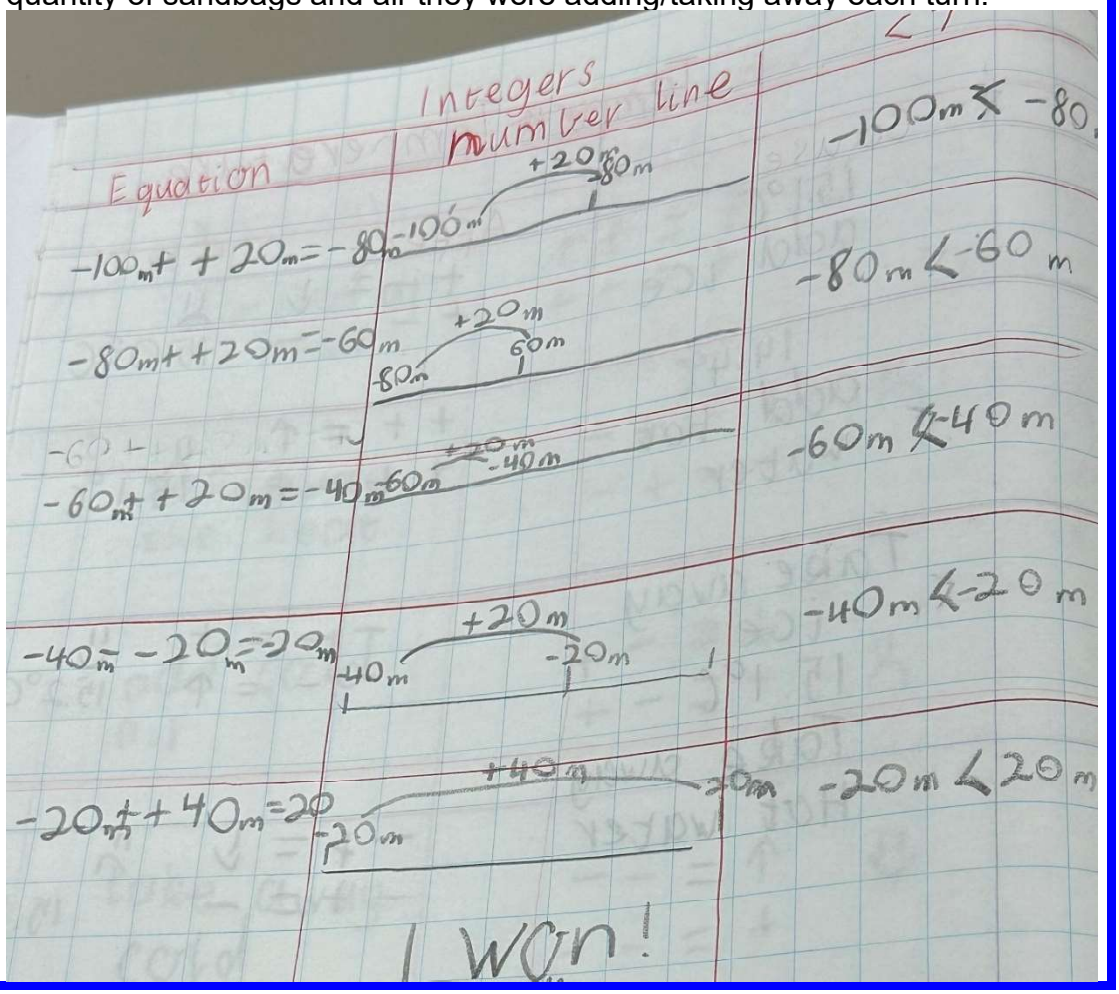
For example, let's say the student who pulled the elements out of the cup collected the 'add' post-it note and a yellow cube. They imagine placing that extra number of sandbags onto their balloon, which makes it heavier and forces it down the number line, because they have added sandbags. For real-life context purposes, imagine you are racing other balloons and a few competitors throw sandbags onto your balloon from theirs – yours would go down!

Questioning:

- When making your number line, where should zero go? Where should the positive and negative numbers go?
- Is zero (**the origin**) a positive or a negative number? Explain why. (Neither).
- Which number is lower: -12 or -15? Think about it using this lesson's context. Would you prefer your balloon to be -12cm under the water, or -15cm? Discuss your opinions with your partner, then share as a class. *Example reasoning:* -15 is lower and more negative than -12, because it is further away from 0 (the surface of the water), and is further underwater. Therefore, -12 > -15 (negative 12 is greater or higher than -15).



Student recording throughout the game: Most students used vertical number lines in the centre column, but some preferred to use horizontal, as shown below. Some students set up number lines in intervals of 10m, rather than by ones, rolling a tens place value dice instead of a 3-dot dice for the quantity of sandbags and air they were adding/taking away each turn:



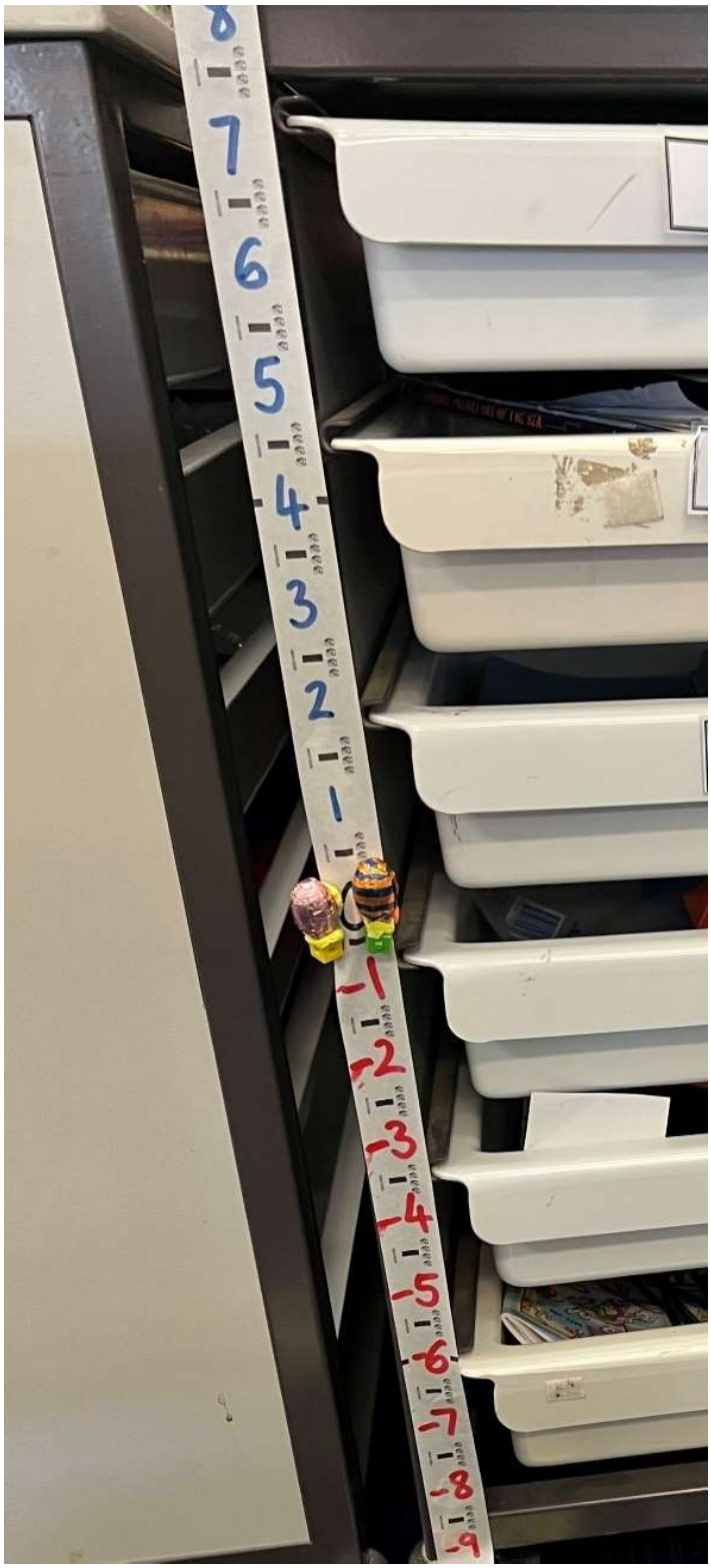
One student pulls from the cups, as the other records their last turn. It is critical for students to record, as this connects the abstract equations (+ + = +, - + = - , - - = +) to the real-life context within the game.

Game in action – video link

Game in action – video link

https://www.youtube.com/watch?v=q9InWsYTlYo

Game in action – movements for the first 2 turns of each student



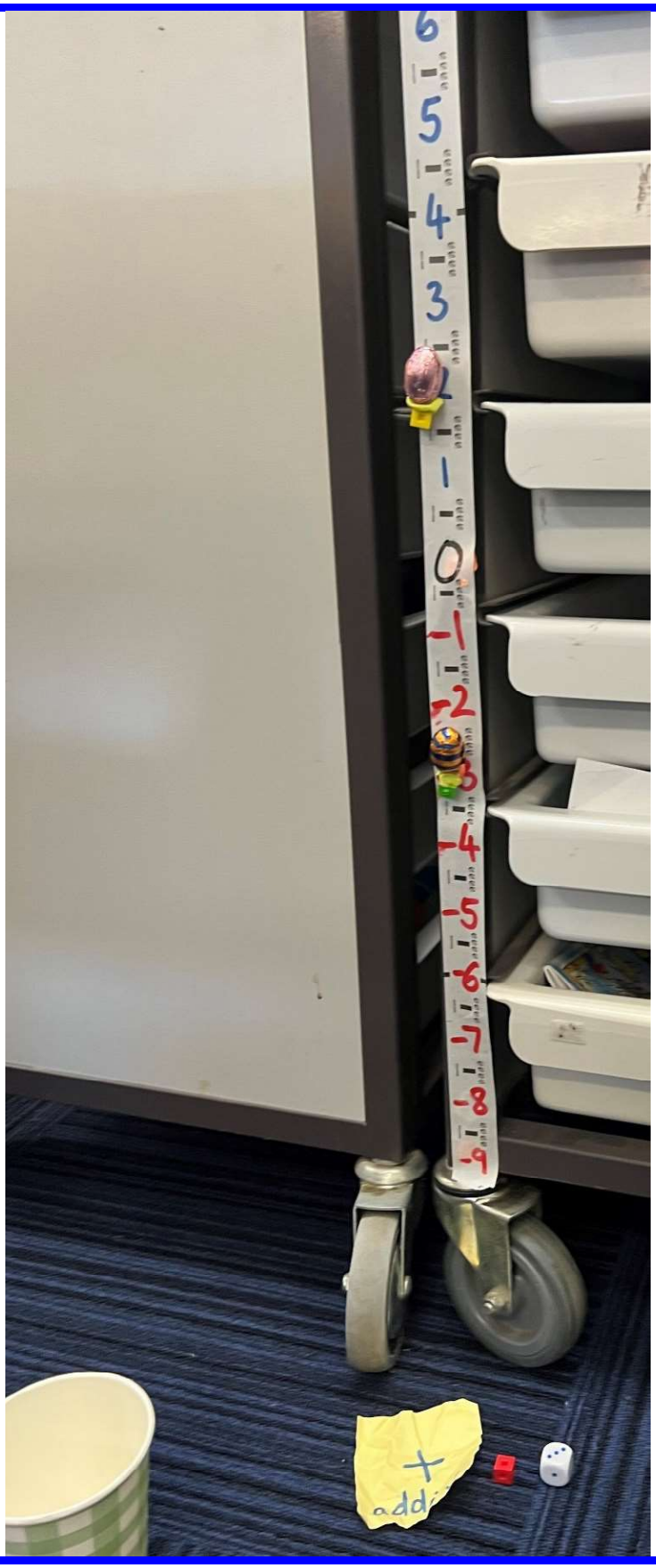
Both balloons start at the '0' mark – balloons were made from Easter Eggs and cubes stuck beneath these, as it was close to Easter at the time of this game.

Student A (pink balloon with yellow base) pulls a ‘take away’ post-it note and red/yellow cube, meaning they need to take away sandbags.

The student reasons that taking away sandbags (- -) would make their balloon lighter/go up (= +), so the balloon moves up/more positive by 2 spaces (the number rolled on the dice was 2).

Equation recorded:

0 - - 2 = +2

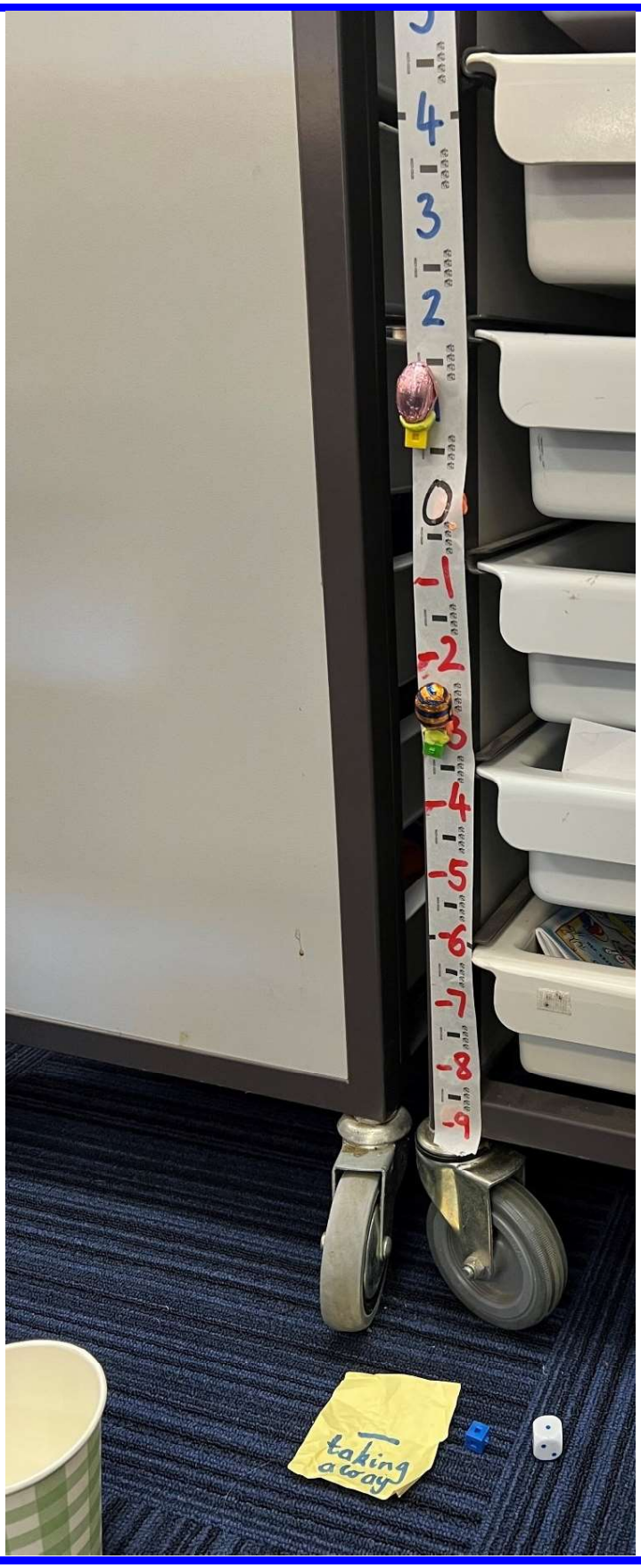


Student B (orange balloon with green base) pulls an ‘add’ post-it note and red/yellow cube, meaning they need to add sandbags.

The student reasons that adding sandbags (+ -) would make their balloon get heavier/go down (= -), so the balloon moves down/negative by 3 spaces (the number rolled on the dice was 3).

Equation recorded:

0 + - 3 = -3



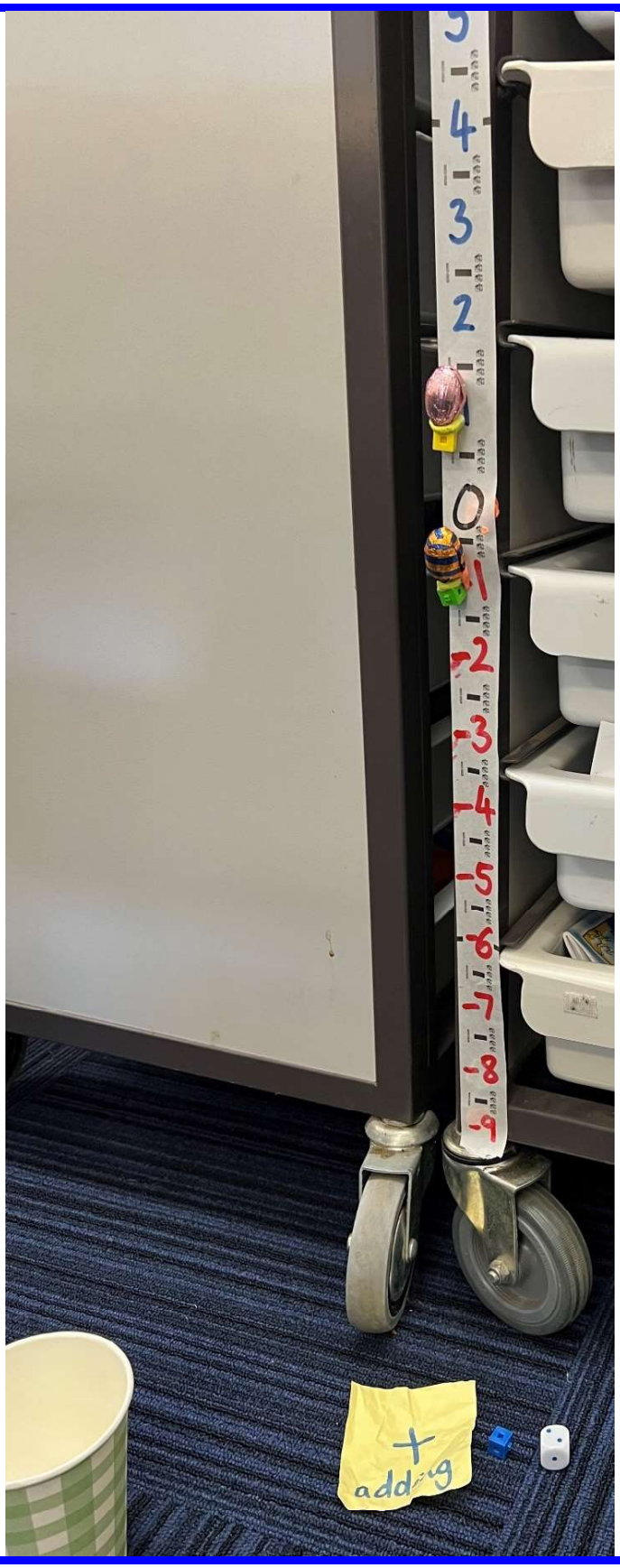
Student A (pink balloon with yellow base) then pulls a 'take away' post-it note and blue cube for their second turn, meaning they need to take away air, or deflate their balloon.

The student reasons that this would make the balloon go lower since you are taking away air (- + = -), so the balloon moved down/negative by 1 space (dice roll was 1).

Equation recorded, starting from student A's last position of +2:

$+2 - + 1 = +1$

"+ 2 take away 1 positive = + 1"



For student B's second turn (orange balloon with green base), they pull an 'add' post-it note and blue cube, meaning they need to add air, or inflate their balloon further with hot air.

The student reasons that this would make the balloon go higher/more positive, since you are adding extra air (+ + = +), so the balloon moved up by 2 spaces (dice roll was 2).

Equation recorded, starting from student A's last position of -3:

$-3 + + 2 = -1$

"-3 add 2 positives = - 1"

Support 1 – for students who are only ready for one symbol/operation, mirroring Lesson 1 to explore the sequence of positive/negative numbers (regular Year 6 content): Eliminate the element of the post-it notes, dice and cubes. Instead, only use one operation, determined by the colour of the playing cards they pull.

Red means the balloon gets more sandbags and goes down (negative). Black means the balloon gets more air and goes up (positive). So, the cards show the **directionality** (which direction the balloon is going), and the number on the card shows the **magnitude** (how much it moves in that direction).

For example, the balloon starts with nothing in its basket, sitting at 0. Students pull a red 6 card (red = negative). Students imagine adding 6 sandbag cubes to their basket, and the balloon falls to -6.

Students record:

Directed number sentence	Number line	Compare to last position
0 – 6 = -6	Encourage students to draw this vertically to match the appearance of the actual number line they are playing on, marking where their balloon was and where it ended up.	0 > -6 *or write:* 0 is higher than -6

To continue the example, students are now at -6 and pull an 8 black card (positive). Add 8 blue counters to their balloon, representing hot air that forces the balloon upwards into the clear blue sky. These effectively counteract the sandbags, so students can get rid of 6 sandbags using 6 blue counters, still left with 2 blue counters.

Students record again:

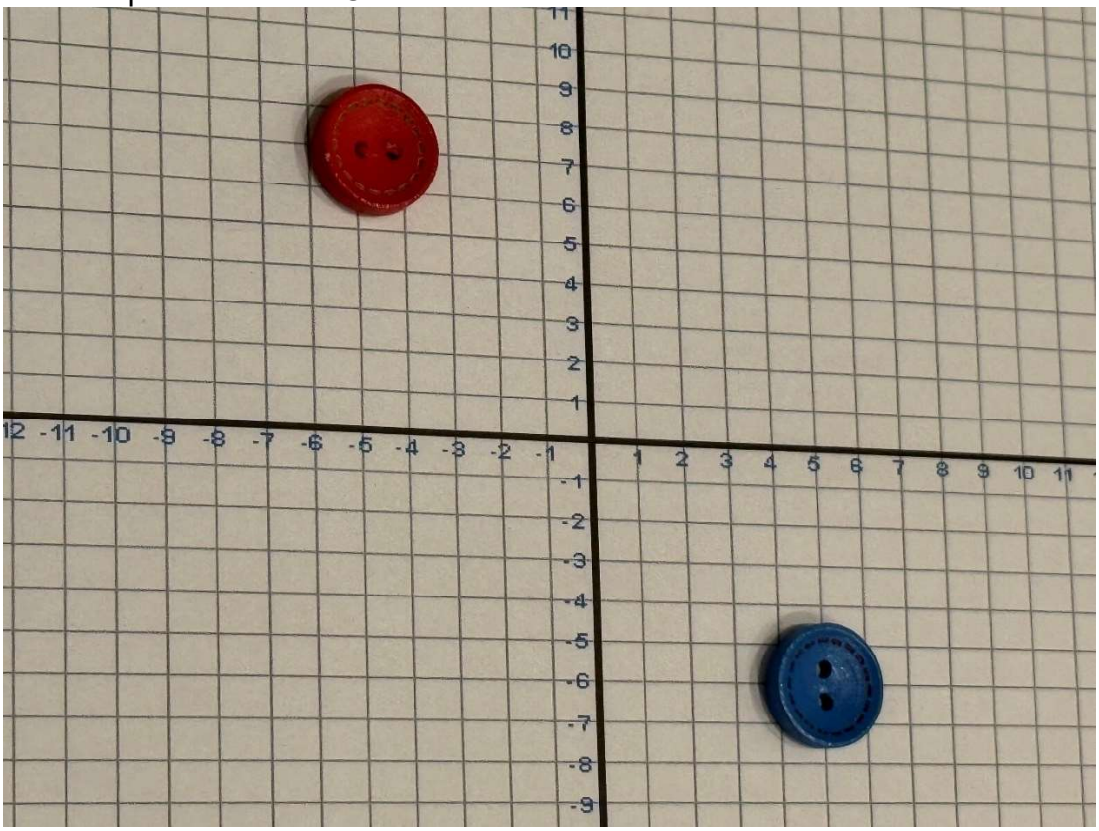
Directed number sentence	Number line	Compare to last position
-6 + 8 = +2	Draw vertically in books	+2 > -6 *or write:* +2 is more than -6

As a final example, students now pull a red 7 (negative 7). Students add 7 sandbags to their balloon, making it more heavy and sink lower down the number line. These counteract the positive 2 blue air counters on their balloon, so 2 sandbags are taken off with 2 air counters. This leaves 5 sandbags, so the balloon sinks to -5. Students continue.

Support 2: Clean their deck of cards so that it only contains 1 to 4 (not 1 to 9). This reduces the mental computation difficulties when mixed with the new concept of negative numbers. Also reduce the range of their number lines to -12 and +12, so the games are shorter and move quicker for more engagement. This also makes the student recording (number line recording column) significantly more supported.

Extreme support: Set up their number line as 0 to +40, adding and subtracting to make it to +40m in the air, but they lose if their balloon reaches 0m. Start at +20m in the air. This removes the negative numbers context from the session and purely focuses on adding and subtracting. Students no longer use sandbags or air counters, but just move their balloon along the number line. Challenge students to solve the addition mentally first, using 10 facts (*Early Years Addition Unit 5*), doubles (*Early Years Addition Unit 6*), near doubles (*EY Addition Unit 7*), building to 10 (*EY Addition Unit 8*), or counting on as the last resort, then check it physically by moving the balloon along the number line.

Mid-range extension – most students can attempt: Set-up the second day of recording on a Cartesian Plane, with the added element of the balloon being pushed 'east' or 'west' by adding these to the post-it note cups. The goal could be to finish closer than your partner to coordinate (+10, +10) in the first quadrant of the Cartesian Plane.



Use a Cartesian Plane template, or rule up one in a grid book.

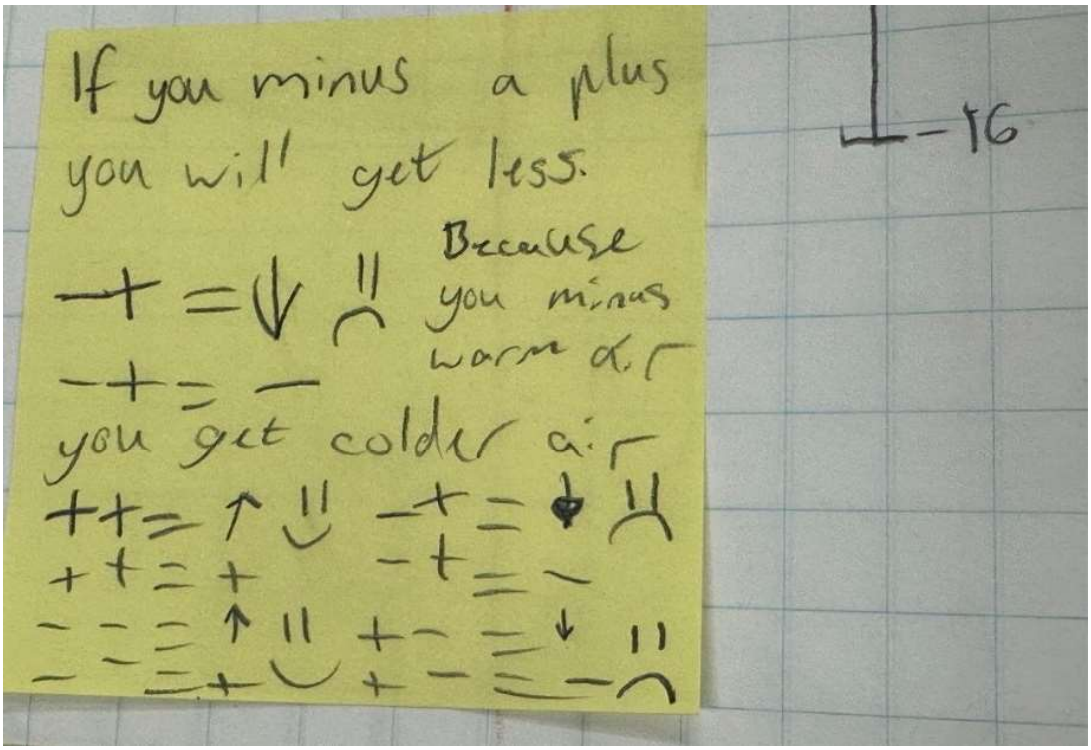
Read about the history of the Cartesian Plane, including the 'fly on the ceiling' story about its origin: https://wild.maths.org/ren%C3%A9-descartes-and-fly-ceiling

Use this interactive at first, or as a checking mechanism for immediate feedback:
https://oercommons.s3.amazonaws.com/media/courseware/relatedresource/file/imth-6-1-9-6-1-coordinate_plane_plotter/index.html

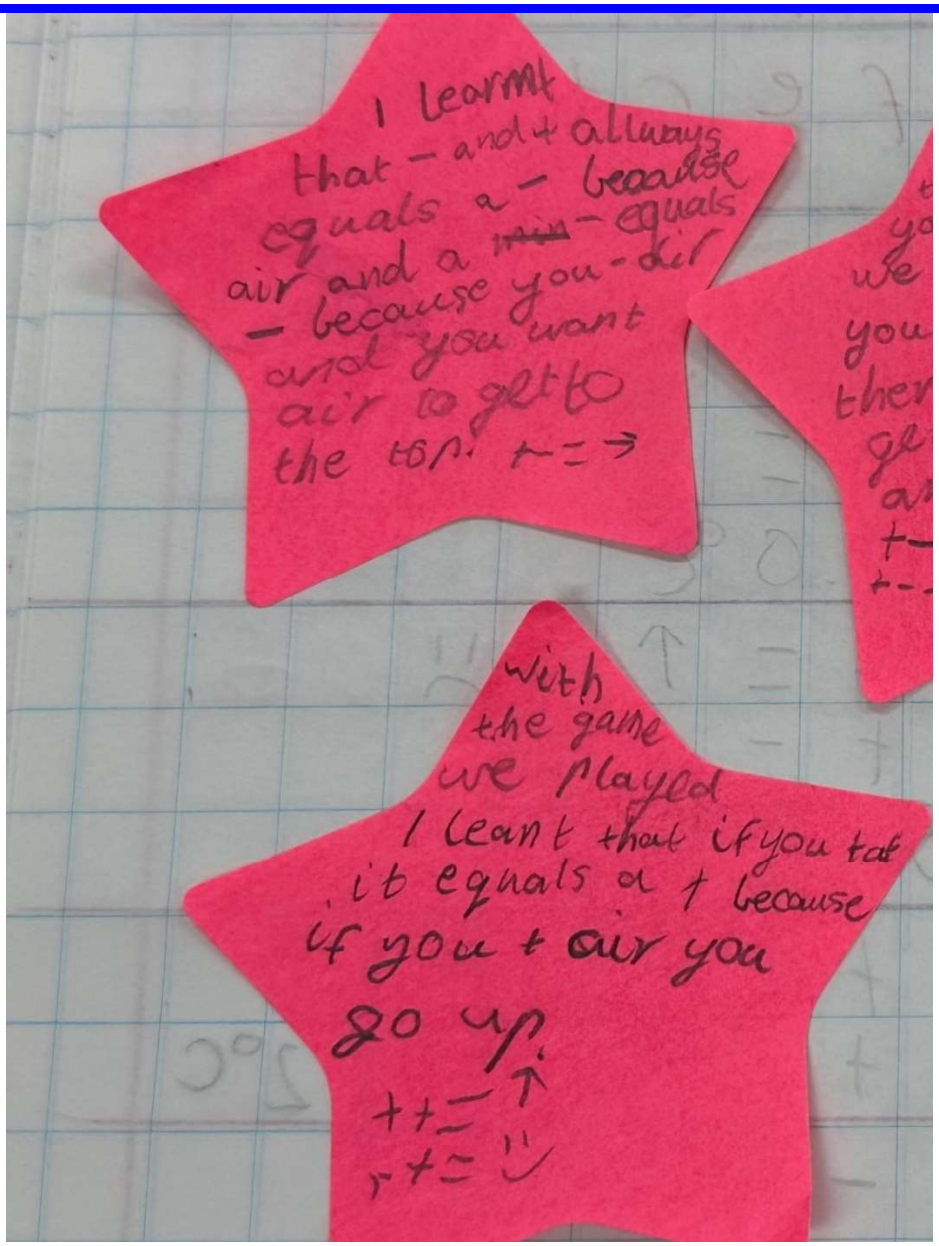
Consider introducing plotting coordinates on the Cartesian Plane using this step-by-step guide:
https://math.libretexts.org/Courses/Kansas_State_University/Your_Guide_to_Intermediate_Algebra/02%3A_Introduction_to_Functions_and_Graphing/2.02%3A_Graphing_on_the_Cartesian_Coordinate_Plane

Extension: Practise using absolute value to work out answers. For example, for -12 + -23, make both answers into absolute values. **Absolute value** just focuses on the distance of the number from zero, or its magnitude. The symbol for absolute value is | |.

Read about absolute value and then try the quiz questions at the bottom of this page. Act out the answers using their balloons on their number lines to check it worked:
https://www.ck12.org/arithmetic/add-integers-with-different-signs/lesson/sums-of-integers-using-absolute-value-msm7/



Reflection post-it note



Reflection post-it notes

Reflection / challenging open-ended question: Why do two negatives make the total more positive? This is one of the trickier questions in mathematics. Have a go at answering it yourself, thinking about real-life contexts involving negative numbers, including the balloon example, or the rotation factor (each negative rotates the character 180 degrees). Tell students you are providing them with a day or two of thinking time, then ask the question again at the conclusion of the next session, requiring examples of explanations and deep reasoning.

Engaging end clip to play during eating time: Hot air balloon designs inspired by movie characters, landmarks and hobbies
https://www.youtube.com/watch?v=lMGZFXi0ZS8&ab_channel=Tooco

Integers
Year 6
Lesson 6

Sea Monster! Or Deep Sea Diver / Sinking Submarine

Learning intention: Add and subtract positive and negative integers, using directed number sentences that include both direction and operation symbols. *Support:* Place integers on number lines and move in both directions on a number line.
Maths vocabulary: integers, positive, negative, add, subtract, compare, direction, magnitude, directed number sentence

Connection to Disney movie Luca – During this session, you are becoming a sea monster, just like Luca! Your parents are trying to send you to the deep ocean to live with Uncle Ugo away from the fun (https://www.youtube.com/watch?v=3AykG8wtLng) but you want to reach the positives out of the depths of the sea to party with the 'land monsters' above the surface, leaping out of the ocean to find your freedom (https://www.youtube.com/

Lesson summary: Students act out a deep-sea dive, adding and subtracting integers represented by weights (negatives), and air that pushes the diver upwards towards the surface (positives).

Video of the game in action

This session is very similar to the Hot Air Balloon Lesson, with a different real-life context and a number line set-up that challenges students to work with negative integers for the entire game.

Materials:

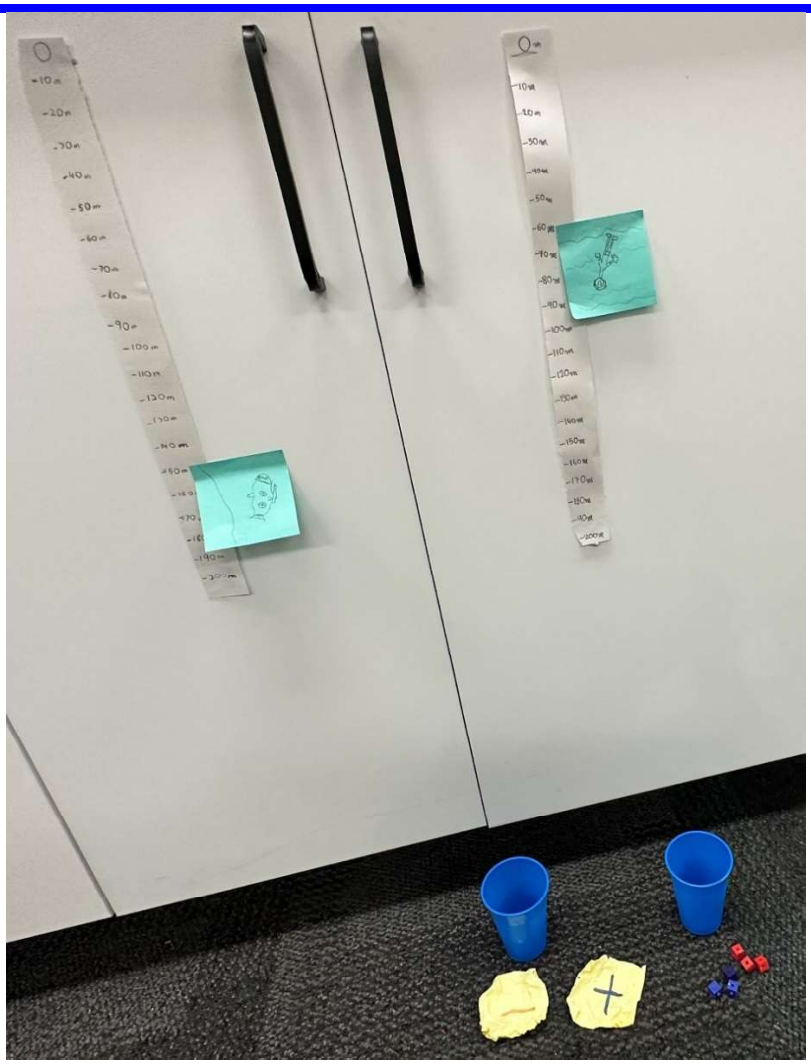
- Deep sea divers, submarines or similar. Students are provided with 5 minutes to create their diver using craft materials such as coloured paper/cardboard, or use Lego figurines or post-it note sketches.
- Thin strips of A3 paper (pre-sliced by the teacher) for students to create a number line that is stuck to one of the legs of their table. At the top, which will be the surface level of the water, write '0m'. Most tables are approximately 1m high, so break the line into 2cm intervals and make the line go down to -50m, which will represent metres under the sea.
- 20-sided dice for 0m to -50m number lines set up in increments of 1m. Place value tens dice (dice with tens on 10 sides = 0, 10, 20, 30, and so on) if setting up their number line from 0m to -200m set up in increments of 10m.
- Red cubes as the diving weights (negatives) and blue cubes to represent the bubbles of water (positives) that push you to the surface.
- 2 post-it notes per pair – one that reads 'add' and another 'take away.'
- 2 cups – one for the cubes, one for the post-it notes.

Best set-up: Tell students the **setting the context story (next page) or use the Luca hook (left-hand side)**. Whole-class model how the game is played at a desk with materials. Model what each cube represents and its meaning, but not its effect – this is for students to discern and investigate to create the rules of ++, -+, +-, --. Students can work individually, as a team, or compete to reach the surface. If they fall to the bottom of the ocean (-50m), they are eaten by a shark!

watch?v=VzcSe4cp15E).

YouTube hook: Scuba dive with one million fish: https://www.youtube.com/watch?v=poF-_IqwG3E&ab_channel=GoPro

YouTube hook: Experience a free dive https://www.youtube.com/watch?v=L4Qm1IehtNQ&ab_channel=NationalGeographic



Video of the game in action

This team set up a 0m to -200m number line as they were using a place value tens dice.

The story goes… "You were diving at -25m and suddenly your air tank starts running out 1% per second. You have limited time to get to the surface. You need to act fast. However, your dive weights keep pulling you down, and the more air your tank loses, the heavier the dive weights feel. Try to make it to 0m (surface level) before your air runs out. If you end up at -50m, there is no way you will make it back – you're doomed!

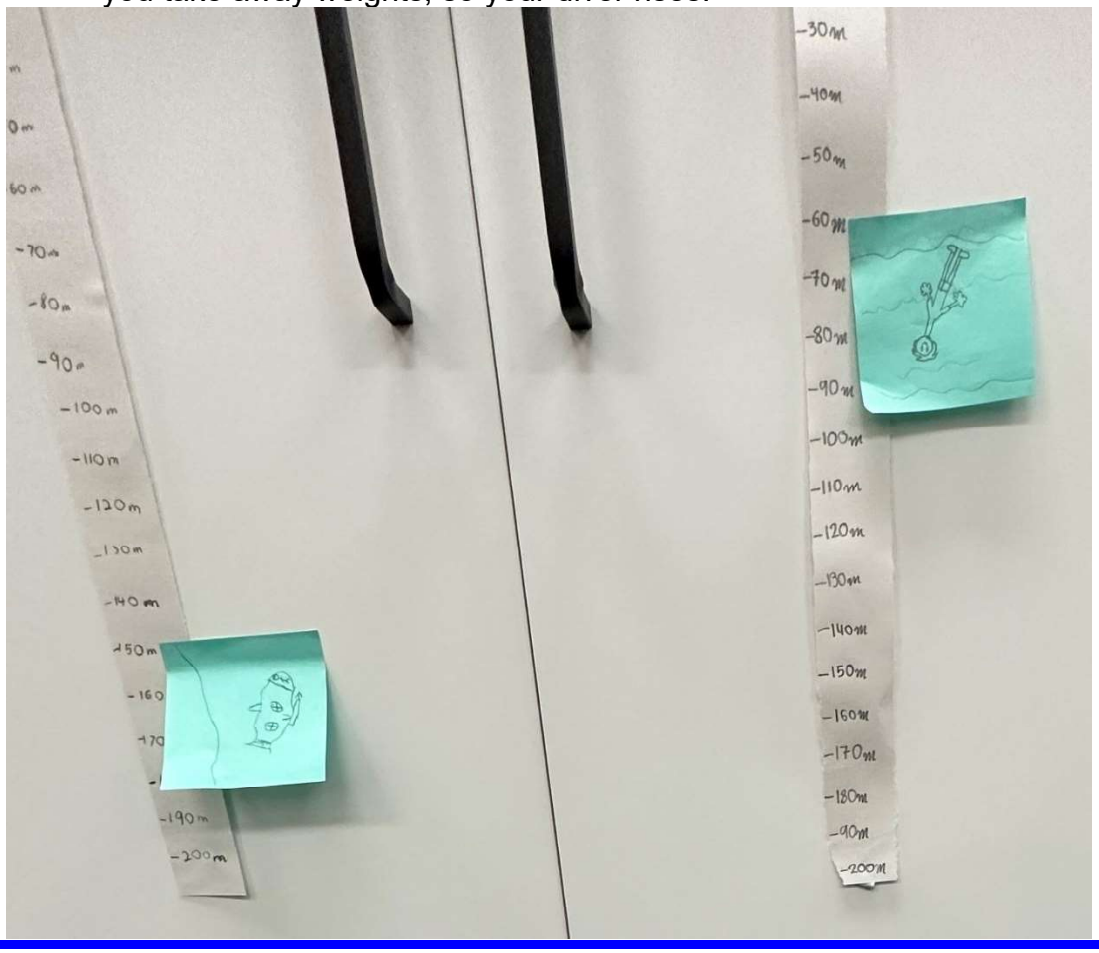
Modelling: Students start their diver in the middle of their number line (for example, if their number line is 0 to -50, start at -25m). The number line is usually best to set up vertically, stuck to the leg of a table.

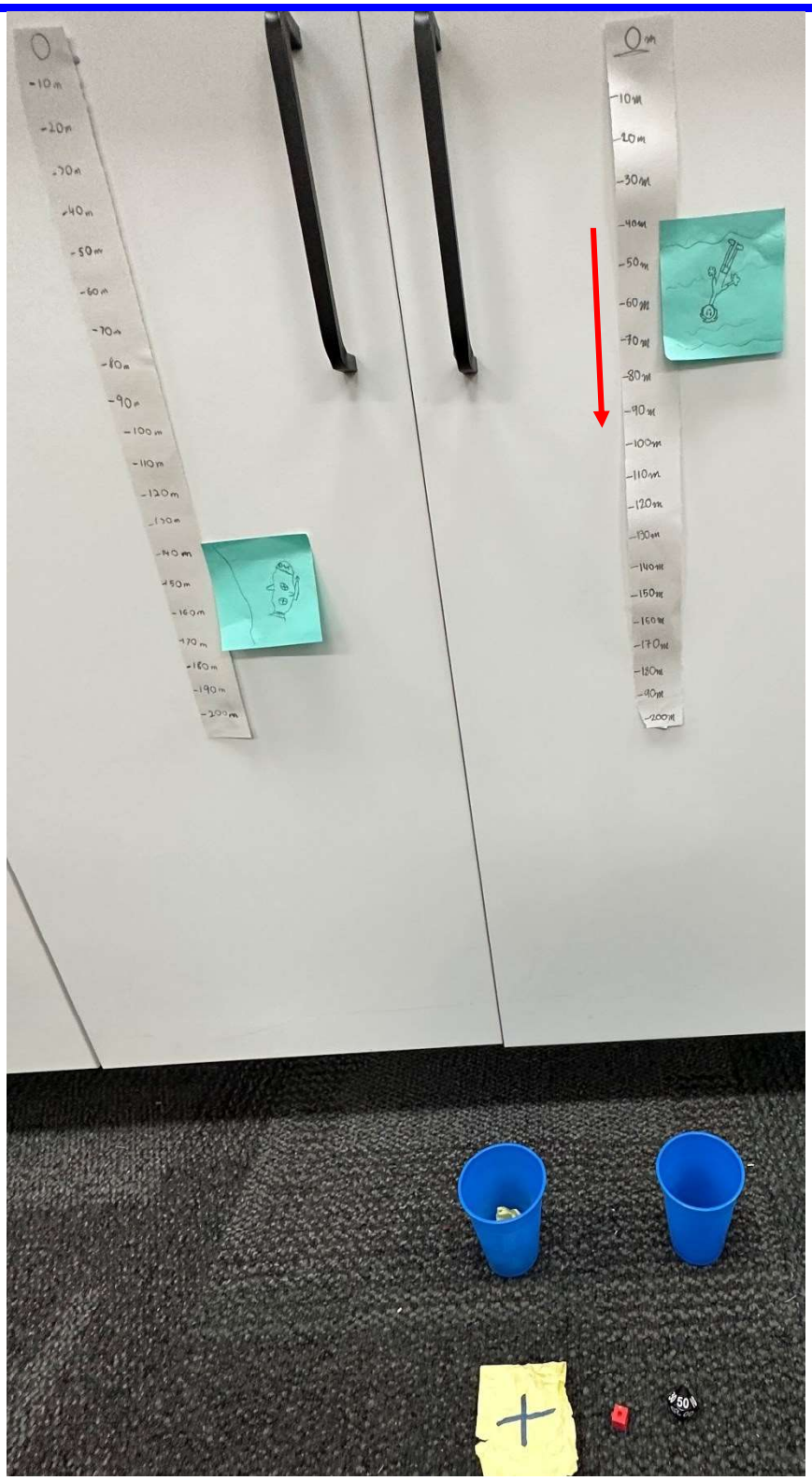
Students pull one element from the post-it note cup and the cubes cup:

- Add = add the item
- Take away = take the item away
- Blue = air, a positive in diving as it makes you rise to the surface
- Red = weight, a negative in diving as it makes you sink to the bottom

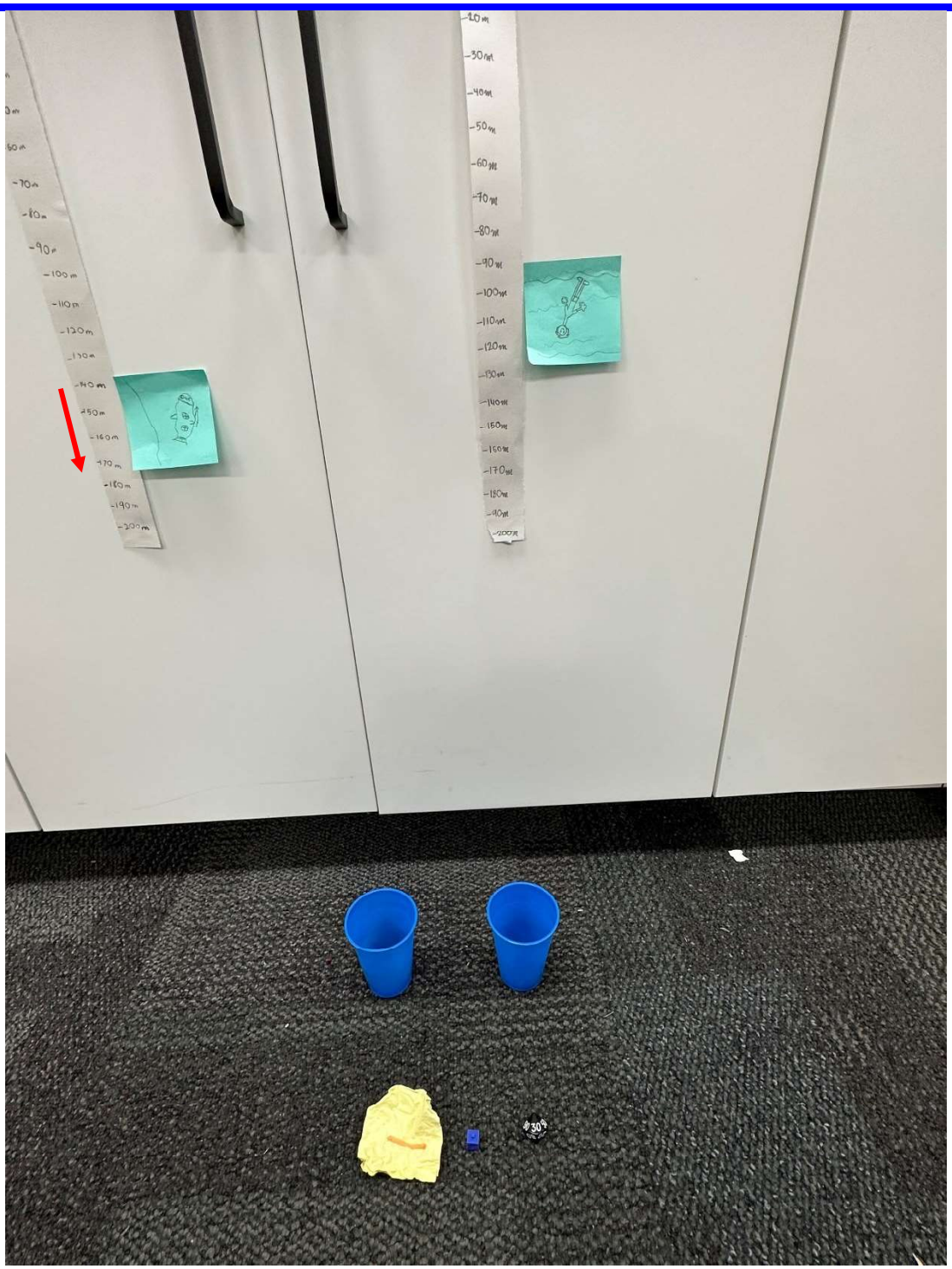
<u>Do not tell these rules to students – these are for them to discern as the game unfolds:</u>

- If you pull 'add' and 'blue cube/air' then that is + +, since you add air, so your diver rises.
- If you pull 'add' and 'red cube/weights', then that is + -, since you add weights, so your diver sinks.
- If you pull 'take away' and 'blue cube/air' then that is - +, since you take away air, so your diver sinks.
- If you pull 'take away' and 'red cube/weights', then that is - -, since you take away weights, so your diver rises.

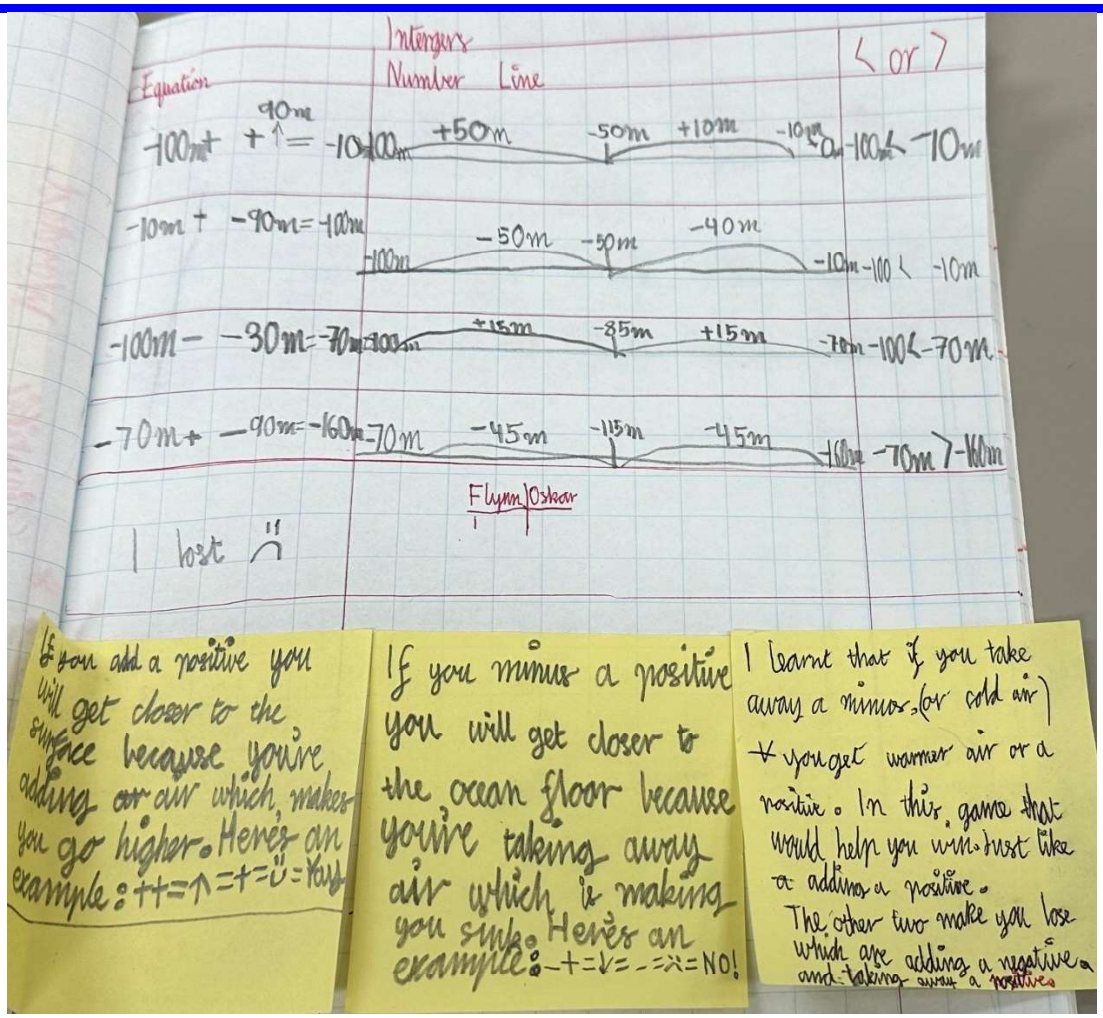




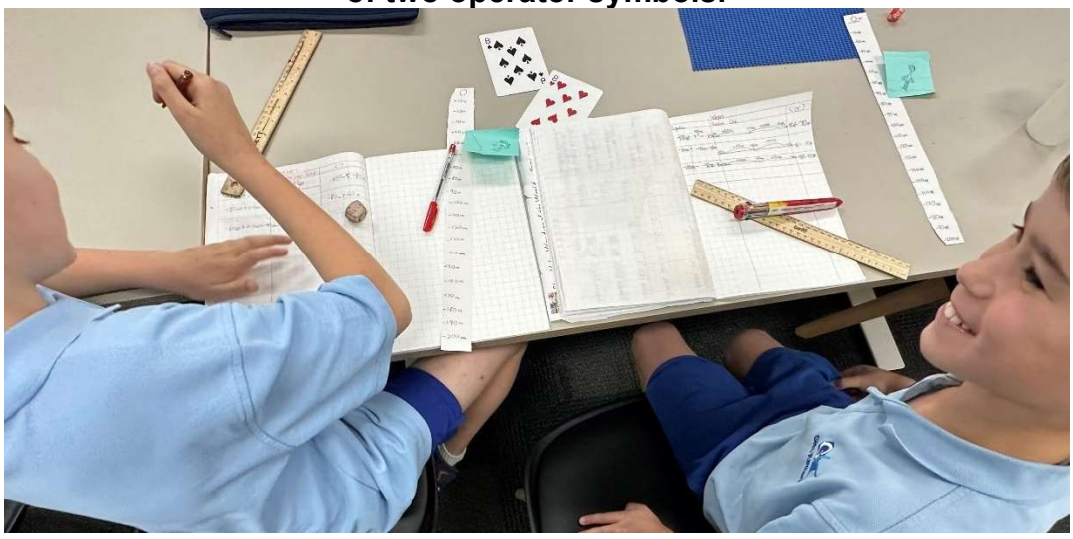
The diver (right-hand player) was on -40m, then pulled out ‘add’ 50 ‘red/weights,’ meaning they sink 50 additional metres, so end at -90m. This is a + - (add weights/negatives), and resulted in a more negative position. The matching equation is -40m + -50m = -90m.



The submarine is on -140m, then pulls ‘take away’ 30 ‘blue/air.’ Taking away air/positive bubbles, makes it sink further to the bottom, so it ends up at -170m. The matching equation is -140m - + 30 = -170m.



Student recording sample with reflection post-it notes after the conclusion of the game summarising what was learned about the effect of two operator symbols.



This game was a real hit with students, who described it as suspenseful and fun.

Game in action

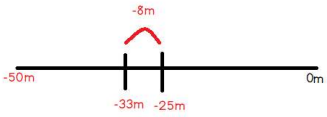
Questioning:

- Which number is lower: -28m or -45m? Think about it using this lesson's context. Would you prefer your diver to be at -28m or -45m below the surface, if you are running out of air? Discuss your opinions with your partner, then share as a class.
- If you are at –35m, how far away are you from the surface? How far up do you need to swim from -35m to reach the surface? That is the additive inverse – the additive inverse of -35 is +35, because they are both an equal distance from the origin and take you back to zero.

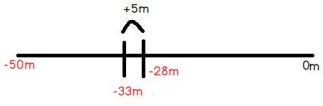
Support 1 – one operator only: Remove the element of two operations. Remove the post-it notes and the dice. Set up the number line from -50 to 0 by ones. Use the cubes to work out the answers to create **zero pairs**, where the air bubbles cancel out the weights they have started with at -25m (start with 25 red cubes).

Students pull playing cards. A red card means they place that extra number of weights onto their diver's belt, which makes her heavier and forces her down the number line, left. A black card means they receive more air to float up to the surface, adding that number of blue counters to their diver. The card colours show the **directionality** (which direction the diver is going), and the number on the card shows the **magnitude** (how much she moves in that direction). Since the diver cannot carry all the counters (as it is just a figurine stuck to the leg of the table), keep track of their weights and air on a grip mat on the desk or floor near the leg of the table.

For example, the diver starts with -25 red cubes, sitting at -25m below the surface. Students pull a red 8 card (red = negative). Students add 8 dive weights (red cubes) to their grip mat below the diver, making 33 red cubes in total, and their diver falls -8 to -33. Students record:

Directed number sentence	Number line	Compare to last position
-25 - 8 = -33		-25 > -33 *or write:* -25 is higher than -33

This student is now at -33 and pull a 5 black card (positive). Add 5 blue counters to their diver, representing air bubbles that force the diver upwards, towards the surface. These effectively counteract the weights, so students can use their 5 air counters to get rid of 5 weights. This leaves 28 red cube weights, or -28m. Students record again:

Directed number sentence	Number line	Compare to last position
-33 + 5 = -28		-33 < -28 *or write:* -33 is lower than -28

Support 2: Clean their deck of cards so that it only contains 1 to 4 (not 1 to 9). This reduces the mental computation difficulties when mixed with the new concept of negative numbers. Also reduce the range of their number lines to -25 to 0, so the games are shorter and move quicker for more engagement. This also makes the student recording (number line recording column) significantly more supported.

Extreme support: Set up their number line as 0 to +40 with a jet pack rider, instead of a diver. Start at 20, adding and subtracting to make it to +40m in the air, but they lose if their jet pack rider reaches 0m. This removes the negative numbers context from the session and purely focuses on adding and subtracting. Students no longer use weights or air counters, but just move their jet pack rider up or down the vertical number line. Challenge students to solve the addition mentally first, using 10 facts (*Early Years Addition Unit 5*), doubles (*Early Years Addition Unit 6*), near doubles (*EY Addition Unit 7*), building to 10 (*EY Addition Unit 8*), or counting on as the last resort, then check it physically by moving their jet pack pilot up or down the vertical number line.

Reflection challenge – why do two negatives make a positive?

Why do two negatives make the total more positive? This is one of the trickier questions in mathematics. Have a go at answering it yourself, thinking about real-life contexts involving negative numbers from the tasks so far.

Great explanations can be found here: https://nrich.maths.org/9958

- Another good context to think about is a submarine. The submarine is at -50m. It is trying to follow another submarine. The other sub surfaces, so it must as well. The helmsman quickly takes away -50 metres of depth, bringing the submarine back to the surface (0m). Essentially, -50 - - 50 = 0 (-50m take away negative 50m, made 0m).
- If you owe a friend some money, by paying it back, you are essentially paying down your debt. Taking away a debt makes you 'back in the black,' which means you no longer owe them money (or as much money) because you subtracted your debt.
- The elevator you are in is overloaded with people (a frequent occurrence in China, where the elevator alarms are often set off by overcrowding). Somehow, you fit in too many and you are going down fast! Luckily, the emergency button stops it and the doors open at a floor. People rush out. However, the doors close too quickly for you to get out. When the doors close, the elevator isn't overloaded anymore – enough people got off, and it starts moving back up to where it was supposed to be going. By subtracting a negative (taking off the weight of lots of the people riding the elevator, which was weighing it down) the elevator became less heavy and more positive.

